



universität
wien

DIPLOMARBEIT

Titel der Diplomarbeit

“Hospital Mergers and their Consequences on the
Hospital Market“

Verfasserin

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angestrebter akademischer Grad

Magistra der Sozial- und Wirtschaftswissenschaften
(Mag.rer.soc.oec.)

Wien, im März 2009

Studienkennzahl lt. Studienblatt: A-140
Studienrichtung lt. Studienblatt: Diplomstudium Volkswirtschaft
Betreuer: Univ.-Prof. Dr. Maarten Janssen

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1 Introduction

From the late 1980s hospital merger activity has increased particularly in the United States¹. In Europe, the public interest to provide a nationwide health care system with blanket coverage, has resulted in a large number of relatively small hospitals and an oversupply of hospital beds in some countries. This applies especially to Austria and Germany, where a rate of 6,1 and 6,4² beds per 1.000 population is well above the OECD average of 3,9³.

Both considerations on quality and cost efficiency in public healthcare have sparked the interest to bundle hospital activities⁴. Especially in Europe, where the vast majority of all hospitals are not-for-profit ones⁵, mergers have the reputation to cut the costs and hence reduce the financial strain on the public healthcare system. Consequently, mergers in the hospital sector and the development of hospital associations are becoming very popular in the European Union.

While a lot of literature on hospitals in the United States is available, only a small number of scientific papers can be found on European hospital markets. In particular the availability of literature on estimated hospital cost functions and hospital mergers in Europe is limited. Consequently, any investigation on the effects of mergers on the European hospital market or potential economic incentives for hospitals or hospital wards can not be supported by corresponding scientific data.

In addition to the situation described above, the special characteristics of a merger analysis of the hospital sector provided yet another incentive for the present work: In traditional merger analysis any standard industry sector aims at maximizing profits in a market, which is normally characterized by price competition. The industry output is clearly defined and easily countable.

However, these general conditions do not apply to the hospital sector, where the units of output can neither be explicitly defined nor easily measured. Moreover, due to the fact that for hospitals profit maximization is subject to strict terms of payment – as agreed with public or private health insurers – quantity competition occurs. This statement is particularly valid for the EU health sector, which is predominantly comprised of not-for-profit hospitals.

The focus of this paper is an investigation of possible consequences of mergers between hospitals or hospital wards on the hospital market. In order to illustrate the effects of a merger on costs and profits of the insiders and the outsiders a merger analysis is being conducted. Furthermore, the implications of mergers will be discussed for the private- as well as the public hospital market. The question whether a price increase is the inevitable result of a merger will be answered for both cases.

In order to address the issues at hand in sufficient detail, the content of this paper is structured as follows:

¹ See Sinay U.T.: Pre- and Post-Merger Investigation of Hospital Mergers (1998)

² Wörz M., Busse R.: Analysing the impact of health-care system change in the EU member states – Germany (2005)

³ Köck C.: Krankenhäuser können gefährlich sein (2009)

⁴ See Köck C.: Krankenhäuser können gefährlich sein (2009)

⁵ Wörz M., Busse R.: Analysing the impact of health-care system change in the EU member states – Germany (2005)

In the next chapter traditional horizontal merger analysis (i.e. horizontal mergers between single product firms) is introduced. A comparison of the one-shot non-cooperative equilibrium in the industry before and after a horizontal merger is given.

Since hospitals are multiproduct firms producing a wide range of different outputs, chapter 3 provides a brief recapitulation of the basic concepts of multiproduct cost functions in general, and of economies of scale and scope in particular.

It is often difficult to measure different outputs from a service sector such as the hospital market. Moreover, hospital outputs can be regarded as being the most important variables to be included into a hospital cost function. Thus hospital outputs, their units and method of measurement are defined in chapter 4.

Once the basic parameters for the hospital sector have been identified, the general layout of a hospital cost function together with all variables to be included therein is analyzed in chapter 5.

On the basis of the conclusions drawn in the previous chapters an adequate hospital cost function for the purpose of calculation is defined in chapter 6 and a merger analysis of the hospital sector is conducted. The main focus of this analysis is on the changes in costs, profits and prices, which might occur as a result of a merger. Though the general approach is similar to the traditional merger analysis described in chapter 2, the special characteristics of the hospital sector have to be observed. Hence the multiproduct case instead of the single product case is analyzed.

In the course of the merger analysis presented in this work it will be shown, that for a public hospital market mergers between hospital wards or hospitals are always beneficial – irrespective of any price increase – since the joint profits of the total market are higher after a merger than before. In contrast the results indicate that in a private hospital market only merging hospital wards or hospitals will benefit, while all other market participants will suffer financial losses.

2 Traditional Horizontal Merger Analysis

Analyzing the effects of a horizontal merger, economists traditionally focus on single product firms. Due to this fact I want to give a brief review of the single product case to highlight the differences to the main topic of this thesis, horizontal merger analysis of multiproduct firms like hospitals, dealt with in detail in chapter 6. Supposedly Cournot competition is a good way of describing the hospital market where it is more likely that hospitals compete in quantities and not in prices. This applies especially in Europe, where a broad public health insurance system is available. Consequently, an analysis of horizontal mergers in the hospital market is given in chapter 6 whereas chapter 2 describes mergers of one product firms in a market where Cournot competition is present.

The subject of this chapter is the comparison of the one-shot non-cooperative equilibrium in the industry before and after a horizontal merger. In doing so, one has to distinguish between mergers which induce efficiency gains and between mergers where these gains are absent.

2.1 Horizontal Mergers which Create no Efficiency Gains

In his book Motta⁶ states that it is likely that a merger increases the market power of the merging firms without creating any efficiency gains. This is accompanied by a reduction of the consumer surplus and the total welfare.

In the Cournot competition scenario firms determine the output quantities they are going to produce. If a merger takes place, the merging firms (i.e. insiders) reduce their output quantities to raise the price charged for this product, whereas the other firms in the market (i.e. outsiders) expand their production which will then reduce the price increase. J. Farrell & C. Shapiro⁷ formulate these findings in their lemma⁸ stating that under popular assumptions⁹ the aggregate output moves in the same direction as the insider's output, but on a smaller scale. This supports the comment of Motta and also of M. Perry & R. Porter¹⁰, who mention that with absent efficiency gains the outsiders will benefit more from the merger than the insiders. In addition, M. Perry & R. Porter conclude that this is also the reason why an increase in total industry profits does not need to be profitable for the insiders.

Furthermore, they state that the profits of the insiders can only exceed those of the constitute firms in the pre-merger situation if the merger indicates a price increase large enough to offset the reduced output.

M. Perry & R. Porter suggest that if the number of insiders is small the output reduction associated with the merger is also small, indicating a small price increase and reducing the profitability of the merger. Consequently, there exists an incentive for a large number of firms to merge but not for a small number.

⁶ Motta M.: Competition Policy: Theory and Practice (2003)

⁷ Farrell J., Shapiro C.: Horizontal Mergers: An Equilibrium Analysis (1990)

⁸ Analyzing a homogeneous-goods industry

⁹ The firms' reaction curves are downward sloping and each firm's residual demand curve intersects its marginal cost curve from above.

¹⁰ Perry M., Porter R.: Oligopoly and the Incentive for Horizontal Merger (1985)

S. Salant & S. Switzer & R. Reynolds¹¹ demonstrate in their paper¹² that a merger among symmetric firms is unprofitable if less than 80% of the firms in the industry merge. They also show that a merger to a monopoly is always lucrative because then joint profits will be maximized. Furthermore they state that “(...) if a merger by a specified number of firms causes losses (respectively gains), a merger by a smaller (larger) number of firms will cause losses (gains).”

J. Farrell & C. Shapiro mention that a merger without synergies will result in a price increase. They set up the proposition that in a Cournot oligopoly a price increase occurs only if the mark-up of the insiders is less than the pre-merger mark-ups of the constitute firms. This statement does only apply if the insiders output is as much as the aggregate output of the constitute firms in the pre-merger situation. Furthermore, the authors note that if under certain assumptions¹³ a merger among firms is profitable and raises the industry price it would although raise welfare.

In addition, Motta recognizes that a merger even without efficiency gains can increase welfare if the insiders consist of small firms whereas the outsiders are large ones. It is feasible that an output reduction of the small merging firms induces a larger output expansion from the outsiders.

In summery, the traditional model in industrial economics as given above indicates that a merger without creating any efficiency gains is likely to reduce consumer and total welfare, raises the industry price and the producer surplus and also increases the market power of the insiders.

After describing the horizontal merger analysis with absent efficiency gains the results shall be illustrated by a sample calculation¹⁴.

2.1.1 Sample Calculation

In the simplest and also most popular standard Cournot model n firms produce perfectly homogeneous goods with constant marginal costs c . Capacity constraints do not exist so the firms are able to satisfy all the demand they face.

Suppose the market demand is given by the inverse demand function $p = a - Q$ where Q represents the total industry output.

– Pre-Merger Situation

The equilibrium output of any of the $i = 1, \dots, n$ firms in the industry can be calculated as follows:

$$\max_i \pi_i = (a - Q - c)q_i = (a - \sum_{j=1}^n q_j - c) q_i \quad (2.1)$$

¹¹ Salant S., Switzer S., Reynolds R.: Losses from Horizontal Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium (1983)

¹² As J. Farrell and C. Shapiro (1990) the authors also analyzed a homogeneous-goods industry.

¹³ The merging firms have sufficiently small market shares, p'' , p''' and their marginal costs are non-negative. The third derivative of their cost functions with respect to their quantities is non-positive.

¹⁴ The exercise and also the solution can be found in Motta M.: Competition Policy: Theory and Practice (2003)

To maximize the profit function of any of these firms, the first order condition has to be calculated:

$$\frac{\partial \pi_i}{\partial q_i} = a - 2 q_i - \sum_{j \neq i} q_j - c = 0 \quad (2.2)$$

Because the products of the different firms are perfectly homogeneous one can insert $(n - 1)q_i$ instead of $\sum_{j \neq i} q_j$ into the first order condition of the i^{th} firm. This gives

$$a - 2 q_i - (n - 1)q_i - c = 0 \quad (2.3)$$

Solving this equation for q_i , the equilibrium output for any firm in the industry can be obtained:

$$q_i(n)^* = \frac{a-c}{n+1} \quad \forall i = 1, \dots, n \quad (2.4)$$

Inserting the equilibrium output into the profit function above gives the equilibrium profit from one firm in the industry:

$$\pi_i(n)^* = (a - n \cdot q_i^* - c)q_i^* = \frac{(a-c)^2}{(n+1)^2} \quad \forall i = 1, \dots, n \quad (2.5)$$

– Post-Merger Situation

Suppose that $m + 1$ firms merge and that because of the standard assumptions in the oligopoly model (homogeneous goods, no capacity constraints) there are now m firms less in the industry. Therefore, the industry consists of $n - m$ independent firms after the merger.

Following the calculation principle from the pre-merger situation¹⁵, one can obtain the equilibrium output of any of the $n - m$ firms in the industry:

$$q_i(n - m)^* = \frac{a-c}{n-m+1} \quad \forall i = 1, \dots, n - m \quad (2.6)$$

By inserting again the equilibrium quantities into the profit function the equilibrium profit of one firm in the industry can be obtained:

$$\pi_i(n - m)^* = \frac{(a-c)^2}{(n-m+1)^2} \quad \forall i = 1, \dots, n - m \quad (2.7)$$

An outsider always gains from the merger if $\pi_i(n - m) > \pi_i(n)$ holds. For the insiders a gain from the merger can be found if $\pi_i(n - m) > (m + 1) \cdot \pi_i(n)$. This is due to the fact that in the pre-merger situation $(m + 1)$ firms which are then involved

¹⁵ The single difference being $n - m$ instead of n firms in the industry

in a merger obtain the equilibrium profit whereas after the merger only one of the $(m + 1)$ firms persists.

The merger situation can also be shown graphically. Since in the Cournot model quantities are strategic substitutes, the reaction functions of the firms are downward sloping.

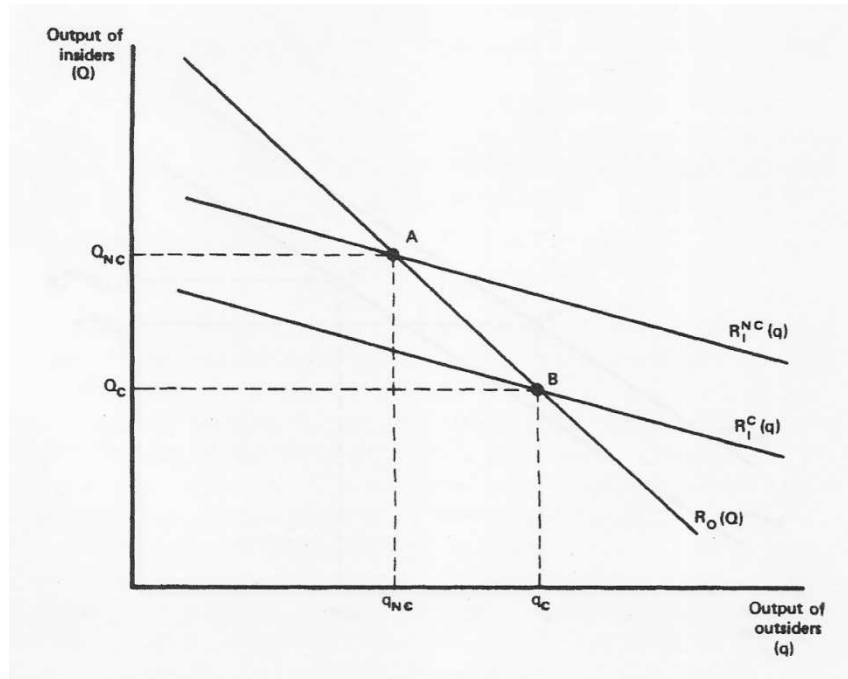


Figure 1 Production Responses of a Merger with no Efficiency Gains, Strategic Substitutes
Salant S., Switzer S., Reynolds R. (1983)

The equilibrium in the pre-merger situation is given by the point A in the graph, where the future insiders produce the quantity Q_{NC} ¹⁶ and the outsiders turn out q_{NC} . After the merger the insiders restrict their aggregate output, internalizing the losses caused to each other. For that reason their reaction function shifts downward into the new equilibrium position B.

In the post-merger scenario the insiders produce an aggregate quantity of Q_C ,¹⁷ and the outsiders produce q_C . As already mentioned in the Cournot model, quantities are strategic substitutes which mean that the rivals' response to a quantity restriction of the insiders will be a quantity expansion. The outsiders then have the possibility to gain market shares but they simultaneously diminish the price increase from the insiders. Consequently, the insiders lose market shares and in addition have a profit reduction if the lower quantity produced is not compensated through the price increase in the market (i.e. if the outsiders expand their production too much).

The profit situation can also be shown in a graph:

¹⁶ NC stands for noncolluding insiders.

¹⁷ C stands for collusive insiders.

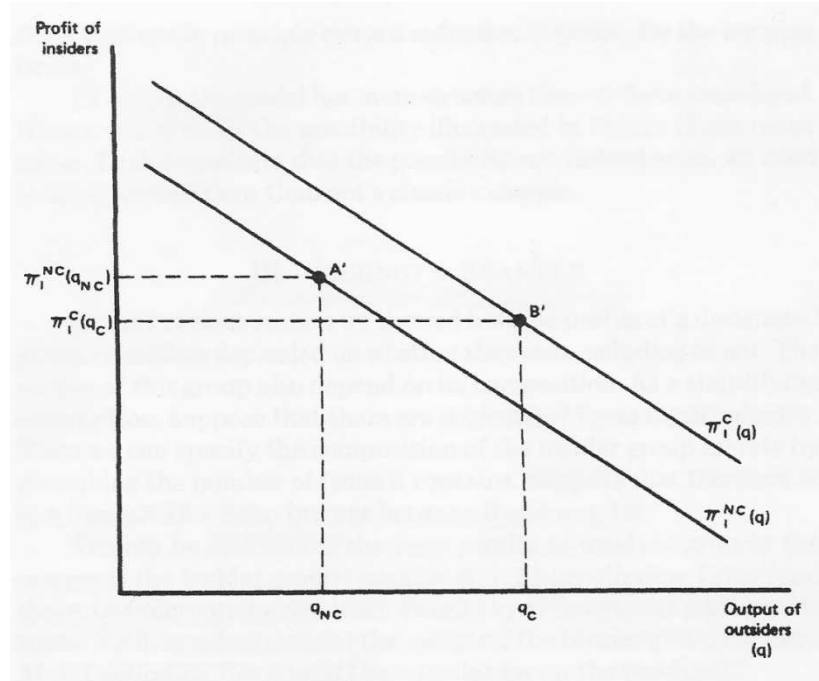


Figure 2 Profit Consequences of a Merger with absent Efficiency Gains
Salant S., Switzer S., Reynolds R. (1983)

S. Salant & S. Switzer & R. Reynolds¹⁸ state that irrespective of the output quantities the rivals are going to produce, the aggregate profits of the insiders have to be higher in the post-merger situation than in the pre-merger one since they can always adopt their production to copy the pre-merger situation. This is illustrated in figure 2 with the line $\pi_i^C(q)$ ¹⁹ situated above the line π_i^{NC} . The authors note that it is somewhat surprising, that even under this condition losses due to horizontal mergers are possible. The reason for this occurrence is the fact that the equilibrium in the pre-merger situation disappears after a merger so that the insiders have the incentive to restrict their output quantities²⁰.

In the pre-merger situation the equilibrium profits of the insiders are represented by the point A'. It is possible that after the merger the outsiders increase their production to an extent that the aggregate profits of the insiders will fall (this scenario is illustrated in figure 2). The new equilibrium profits of the insiders are now represented by point B'. As can be seen in the figure and has been mentioned above, the profitability of a merger can not be regarded as a certainty.

Looking at mergers creating efficiency gains the situation described above changes. This problem will be addressed in the next section.

¹⁸ Salant S., Switzer S., Reynolds R.: Losses from Horizontal Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium (1983)

¹⁹ I represents the insiders.

²⁰ Given that the output quantities of the rivals are unchanged.

2.2 Horizontal Mergers which Induce Efficiency Gains

As mentioned in the previous section, a merger which does not coincide with efficiency gains is expected to reduce consumer welfare and welfare in total. The situation differs if gains in efficiency due to a merger can be achieved. If the insiders become more efficient (resulting in lower unit costs), they can outweigh the increase in market power. If the cost reduction is large enough, prices can fall and consumers will benefit from the new industry structure. The reason for this development is that a reduction in output and thus a price increase may not be the most profitable strategy if efficiency gains are involved. Reducing prices and attracting new customers can be a more lucrative strategy. Motta²¹ describes the case where prices and unit costs decrease proportionately after the merger and therefore the unit mark-up remains the same as in the ex-ante case. In the post-merger situation, total profits will still be higher since the lower prices attract new consumers.

If efficiency gains accompany the merger, the insiders have two possible strategies to increase their profits: To reduce the quantities they produce and thus increase their prices or increase produced quantities to reduce the prices charged. Motta states, that the higher the efficiency gains achieved, the more likely the insiders reduce their prices resulting in higher consumer and total welfare²².

J. Farrell & C. Shapiro²³ also analyze a merger situation where synergies²⁴ occur. They find that “rather impressive synergies²⁵ are typically necessary for a merger to reduce price.” Furthermore, they note that the larger the market shares of the insiders or the smaller the elasticity of demand, the greater must be the synergies in order to induce falling prices. In addition, the authors consider that their result is strengthened if a merger changes the competitive conduct from Cournot to something less competitive.

A merger which comes along with gains in efficiency will change the competitive positions in the market. Unlike the results in section 2.1, outsiders may loose from the merger because a reduction of the insiders’ unit costs give them the ability to cut their prices. If the efficiency gains are sufficiently large so that the insiders will really cut their prices, the outsiders’ profits will decrease. In this case the effect on welfare is positive.

S. Salant & S. Switzer & R. Reynolds determine that even if a merger creates efficiency gains through economies of scale, losses can still be caused. Nevertheless, mergers which go along with efficiency gains might be beneficial for society even if the insiders run losses.

There are several reasons why insiders might be able to reduce their costs along with a merger: They may reorganize their production by an improvement of the division of inputs or they may achieve economies of scale or/and economies of scope. Motta also lists synergies in research and development, cost savings in

²¹ Motta M.: Competition Policy: Theory and Practice (2003)

²² For a proof see Motta M.: Competition Policy: Theory and Practice (2003)

²³ Farrell J., Shapiro C.: Horizontal Mergers: An Equilibrium Analysis (1990)

²⁴ The authors quote Lawrence White, who wrote that “all merger proposals will promise theoretical savings in overhead expense, inventory costs, and so on; they will tout synergies.”

²⁵ E.g.: learning, economies of scale

administration, rationalization of distribution and marketing activities or the replacement of less qualified managers as possible benefits gained from mergers. Generally, a distinction whether the cost savings mainly affect variable production costs or fixed costs has to be made. If the synergies mainly change variable production costs, the price setting behavior may be modified, whereas the price decision will remain the same if mainly fixed costs are involved. Nevertheless, efficiency gains in fixed costs can have a positive welfare effect because the insiders' producer surplus will rise while the consumer surplus will not change. Motta states that competition authorities shall have a close look if these cost savings could also be gained without a merger (i.e. the savings are not merger-specific) and thus a reduction of market participants is not necessary and desirable.

The following subsection contains a sample calculation²⁶, which differs from the one given above (2.1.1) since now insiders attract efficiency gains because of the merger.

2.2.1 Sample Calculation

Suppose that there are three identical firms in the market, each of which producing a homogeneous good. As in the above example they have constant marginal costs $c > 0$ and the industry demand is given by $p = 1 - Q$ where Q represents the industry output.

– Pre-Merger Situation

The profit function of each of the three firms is given by:

$$\pi_i = (1 - Q - c)q_i = (1 - q_i - q_j - q_k - c)q_i \quad i, j, k = 1, 2, 3 \\ \text{and } i \neq j \neq k \quad (2.8)$$

To obtain the equilibrium output quantities the first order condition has to be calculated:

$$1 - 2q_i - q_j - q_k - c = 0 \quad i, j, k = 1, 2, 3 \\ \text{and } i \neq j \neq k \quad (2.9)$$

Due to the situation that the firms in the industry produce homogeneous goods, one can obtain a symmetric equilibrium output for any firm in the market:

$$q_c = \frac{1-c}{4} \quad (2.10)$$

By inserting the equilibrium quantities into the inverse demand function the equilibrium price will be achieved:

$$p_c = \frac{1+3c}{4} \quad (2.11)$$

²⁶ The sample calculation is given in Motta M.: Competition Policy: Theory and Practice (2003)

Also the equilibrium profit for any of the three firms in the market is now identified via inserting the equilibrium quantities and the equilibrium price into the profit function of each firm:

$$\pi_c = \frac{(1-c)^2}{16} \quad (2.12)$$

– Post-Merger Situation

Suppose that a merger between two of the three firms takes place (i.e. because the firms produce homogeneous goods, there is one firm less in the market but there is still some competition after the merger). Furthermore one must consider, that the insider achieves efficiency gains in terms of production at marginal costs ec with $e \leq 1$, whereas the outsider still produces at marginal costs c .

Motta²⁷ assumes that firms 1 and 2 merge with only firm 1 persisting as the insider in the market. Firm 3 represents the outsider, which has marginal costs c .

The profit functions from the insider and the outsider are given by:

$$\begin{aligned} \pi_1 &= (1 - Q - ec)q_1 = (1 - q_1 - q_3 - ec)q_1 \\ \pi_3 &= (1 - Q - c)q_3 = (1 - q_1 - q_3 - c)q_3 \end{aligned} \quad (2.13)$$

Calculating the first derivative of the profit function with respect to their own quantity, one obtains the first order conditions:

$$\begin{aligned} \frac{\partial \pi_1}{\partial q_1} &= 1 - 2q_1 - q_3 - ec = 0 \\ \frac{\partial \pi_3}{\partial q_3} &= 1 - q_1 - 2q_3 - c = 0 \end{aligned} \quad (2.14)$$

The reaction functions of the firms are given as follows:

$$\begin{aligned} R_1(q_3) &= \frac{1 - q_3 - ec}{2} \\ R_3(q_1) &= \frac{1 - q_1 - c}{2} \end{aligned} \quad (2.15)$$

These functions also define the equilibrium quantities of the insider and the outsider:

$$q_1 = \frac{(1-c)(2e-1)}{3}$$

²⁷ Motta M.: Competition Policy: Theory and Practice (2003)

$$q_3 = \frac{(1-c)(2-e)}{3} \quad (2.16)$$

Motta notes that the outsider only produces a positive amount in the equilibrium if the cost savings are not too large, i.e.

$$q_3 \geq 0 \text{ if } e \geq \frac{2c-1}{c} \quad (2.17)$$

If in addition $c < \frac{1}{2}$, the expression from (2.17) becomes negative, which means that the outsider will always sell products in the equilibrium²⁸.

Inserting the equilibrium quantities into the inverse demand function gives the equilibrium price:

$$p_m = \frac{(1+c)(1+e)}{3} \quad (2.18)$$

Having the equilibrium quantities and the equilibrium price, one can also calculate the profits of the insider and the outsider in the equilibrium:

$$\pi_1 = \frac{(1-c)(2e-1))^2}{9}$$

$$\pi_3 = \frac{(1-c)(2-e))^2}{9} \quad (2.19)$$

Motta notes, that a price decrease after the merger only occurs if the efficiency gains are sufficiently large. A price reduction after the merger happens if $p_m \leq p_c$, giving the following condition:

$$e \leq \frac{(5c-1)}{4c} \quad (2.20)$$

He recognizes that no matter how large the efficiency gains are, there will never be a price decrease if $c < \frac{1}{5}$. This statement can be explained by the fact that the right hand side of expression (2.20) becomes negative which suggests that the inequality will never occur.

One can also investigate whether the merger is profitable or not. Doing so, one has to check if $\pi_1 \geq 2\pi_c$ applies. After some calculations one obtains that this inequality is valid if

$$e \leq \frac{(4(1+c) - 3\sqrt{2}(1-c))}{8c} \quad (2.21)$$

The solution shows that the merger is only profitable if enough cost savings can be achieved.

²⁸ This is due to the fact that the efficiency gain e is always bigger in this case.

After describing the implications of mergers with and without efficiency gains, one will recognize that both cases will result in an increase in the market power of the merging firms. In the next section I want to describe several variables which can influence the likelihood of an increase in the market power.

2.3 Variables which Influence Market Power

There are several variables which influence the possibility of the insiders to increase their market power. Motta itemizes some of them, which shall be briefly reproduced in this section.

- Concentration

The more independent firms are operating in the market after a merger, the less probable a hurtful situation for the consumers will occur.²⁹

A frequently used concentration index is the Herfindahl-Hirschmann Index (*HHI*), which is defined as the sum of the squares of the firm's market share in the industry³⁰. The values of the *HHI* can vary between 0 (if the market is completely fragmented) and 10.000 (if only a single firm is in the market). An additional measurement for the presumable change in concentration is ΔHHI , which measures the difference between the post- and pre-merger concentration³¹. The European Commission categorizes mergers to be harmless as long as $1.000 \leq HHI \leq 2.000$ if $\Delta HHI < 250$ or $HHI > 2.000$ if $\Delta HHI < 150$ respectively.

- Market Shares and Capacities

As shown above, market shares can also be used as an indicator for a probable change in the market power due to a merger.

J. Farrell & C. Shapiro³² demonstrate in their paper that a small reduction in the insiders output has a net positive welfare effect on outsiders and consumers if the market shares of the insiders are sufficiently small. They also state that the lower the market shares of the insiders the less worse is the effect on the market prices.

Available capacities are also important with regards to possible price changes. The larger the unused capacities of rivals, the less possible it is for insiders to deploy market power.³³ This is caused by the following mechanism: If merging firms raise the prices and consumers have the possibility to switch to rivals (who can naturally satisfy the additional demand) this effect operates as a regulation on the insider's behavior.

²⁹ Other things being equal.

³⁰ $HHI = \sum_{i=1}^n s_i^2$

³¹ $\Delta HHI = \sum_{i=1}^n \sum_{j=1}^n s_i s_j$

³² Farrell J., Shapiro C.: Horizontal Mergers: An Equilibrium Analysis (1990)

³³ Other things being equal.

- Entry

The possibility to increase the prices after a merger is also limited by the presence of potential entrants. If the insiders are going to raise their prices after a merger (i.e. they reduce their output quantities), potential entrants might possibly be activated. Due to this reason, the merging firms may not raise their prices in order to block further market entries. This repression of the market power via potential entrants is primarily dependent on the fixed costs involved. The higher these sunk costs, the higher is the scope for any price increases of the insiders.

- Demand Variables

Demand variables can also affect the market power of a firm. The market may be characterized by high switching costs with consumers not being able to change their provider simply. If this is the case, the merging firms enjoy market power and can therefore raise their prices easier. One can conclude that the lower the elasticity of market demand the higher the range for price increases.

- Buyer Power

It is possible that also downstream firms can restrain insiders which are upstream in the production process. Strong buyers can threaten the merging firms which increase their prices by cancelling orders or by starting the upstream production by themselves.

Looking at the market power which can increase after a merger, one should also take into account the so called failing firm defense. It is feasible that a merger involves a failing firm which can not survive on its own in the industry any longer. In this case, the ex-post merger situation must not be compared with the ex-ante merger situation but rather with the situation that will occur if the failing firm will leave the market.

2.4 Mergers and Collusion

As shown so far a merger can decrease consumer surplus and also total welfare by increasing market power. Generating a better situation for collusion in the market can be regarded as another negative effect of mergers on the welfare. It is possible that before a merger collusion in the industry is not stable, this situation changing when a merger takes place. The reason for this assumption is that a merger reduces the number of independent firms in the market with the collusive behavior of fewer firms needing to be coordinated. It is obvious, that fewer firms in the industry indicate a higher readiness of collusion and also a stronger willingness to charge higher prices. A different pro-collusive effect may be the one that a merger distributes the assets in a certain market more symmetric. If this happens to be the case, a collusive outcome is more likely. Motta also states that it is a priori very difficult to make a statement whether or not a particular merger will lead to collusion.

2.5 Conclusion

Traditional merger analysis predicts that in the single product case horizontal mergers without creating efficiency gains usually cause higher prices and thus lower consumer surplus and total welfare. Nevertheless, also in a situation where no efficiency gains can be obtained the outcome is uncertain. If the quantity reduction of the insiders is more than offset by an output increase of the outsiders, prices will fall and therefore consumer surplus will rise.

The situation after a merger is also ambiguous if efficiency gains can be achieved. A price increase but also a price reduction can reasonably be assumed since new customers can thereby be attracted.

Before investigating the multiproduct case in general and the question how costs and profits will change after a merger in particular, some other general aspects of the topic have to be investigated. The next chapter (3) shall give a brief reminder on multiproduct cost functions, especially on calculating multiproduct scale economies and multiproduct economies of scope. Since hospitals produce special outputs, chapter 4 describes different hospital outputs and their units of measurement. The following chapter 5 contains the basic elements needed for calculation. There different hospital cost functions and the importance of certain variables will be presented. Having found statistically significant variables in hospital cost functions, a hypothesized function will be used for the merger analysis in the final chapter.

3 Multiproduct Cost Functions

Hospitals are multiproduct firms producing a wide range of different outputs. These outputs can be summarized through the following main categories: education, research, community services, outpatient care and inpatient care. Naturally these broad categories can further be subdivided, e.g. into particular diseases, into different education levels or the like.

Before analyzing how to measure these diverse hospital outputs and focusing on hospital cost functions in detail, a brief reminder on the basic concepts of multiproduct cost functions in general, and on economies of scale and scope in particular will be given. Some authors³⁴ state, that economies of scale and scope are present in the hospital sector, which will be discussed in more detail in chapter 5. Hence the present chapter will show how economies of scale and scope can be calculated. Moreover, properties of multiproduct cost functions will be described.

3.1 Properties of Multiproduct Cost Functions

Suppose there are m inputs which are represented by the input vector $x = (x_1, \dots, x_m)$, and n outputs characterized by the output vector $y = (y_1, \dots, y_n)$. The production-possibility set T is given by $T = \{(x, y) \mid y \text{ can be produced from } x\}$ which is simply a list of possible combinations of inputs and outputs.

The multiproduct cost function, where $w > 0$ denotes the vector of constant input prices, is defined as follows:

$$C(w, y) = \min_x \{w \cdot x \mid (x, y) \in T\} = w \cdot x^*(w, y) \quad (3.1)$$

with $x^*(w, y)$ being the vector of input levels that minimizes the cost of producing y at input prices w . In order to have a cost function which is as well positive for positive outputs as it is weakly increasing in output quantities and input prices and furthermore satisfies $C(0) = 0$, a regularity condition is necessary and sufficient:

- Input vectors $x \in X \subset R_+^m$, output vectors $y \in Y \subset R_+^n$
- T is a nonempty and closed subset of $X \times Y$ and
 - $(0, y) \in T$ if $y = 0$
 - $(x, y) \in T, (x', y') \in X \times Y, x' \geq x$, and $y' \leq y$ imply that $(x', y') \in T$

In order to have a well defined multiproduct cost function, further properties have to be fulfilled:

- If $y_i > 0$, $C(w, y)$ has a partial derivative with respect to y_i , denoted by C_i
- $C(w, y)$ is nondecreasing in factor prices w , i.e. $w' \geq w$ indicates $C(w', y) \geq C(w, y)$
- Homogeneous of degree 1 in w , i.e. $C(t \cdot w, y) = t \cdot C(w, y)$ for all $t > 0$
- $C(w, y)$ is concave in w and
- continuous in w

³⁴ E.g.: Sinay U., Campbell C.: Scope and Scale Economies in Merging Hospitals Prior to Merger (1995), Wang J., Zhao Z., Mahmood A.: Relative Efficiency, Scale Effect, and Scope Effect of Public Hospitals: Evidence from Australia (2006)

After this brief reminder³⁵ on the various conditions of cost functions, a detailed view on economies of scale and scope is given in the next section.

3.2 Ray Average Costs and Multiproduct Scale Economies

Average costs in a multiproduct firm, where the production can be described by an output vector $y \neq 0$, are called ray average costs (RAC) and are defined by

$$RAC(y) = \frac{C(y)}{\sum_{i=1}^n y_i} \quad (3.2)$$

Ray average costs are strictly declining at $y = (y_1, \dots, y_n)$ if there exists an $\varepsilon > 0$ so that $\frac{C(vy_1, \dots, vy_n)}{v} < C(y)$ for all $1 < v < 1 + \varepsilon$ and $\frac{C(vy_1, \dots, vy_n)}{v} > C(y)$ for all $1 - \varepsilon < v < 1$. This means that ray average costs are decreasing if a small proportional change in the output brings less than a proportional change in total costs. Furthermore, ray average costs are minimized at y if for all positive $t \neq 0$ $RAC(y) < RAC(t \cdot y)$ applies. As in the single-product case, the theory suggests that the ray average cost curve is U-shaped in the volume of output. This implies that along each ray a unique point, where the ray average costs are minimized, exists.

Ray average costs can also be used to define the degree of ray economies of scale, which measures the reaction of total costs if a proportional change in all output categories takes place (holding all other variables constant).

The degree of scale economies for the whole product set $N = \{1, \dots, n\}$ is given by

$$S_N(y) = \frac{C(y)}{\sum_{i=1}^n y_i C_i(y)} \text{ where } C_i(y) = \frac{\partial C(y)}{\partial y_i} = MC_i$$

$$S_N(y) = \frac{RAC(y)}{MC(y)} \quad (3.3)$$

Economies of scale are increasing ($S_N > 1$), decreasing ($S_N < 1$) or constant ($S_N = 1$) if the $RAC(y)$ are greater than, less than or equal to $MC(y)$.

To identify the product-specific economies of scale is also from particular interest. In this case, one measures the change in total costs for a variation in the output of one product and not of the entire range. In order to do so, it is necessary to introduce the concept of incremental costs. The incremental costs of product $i \in N$ at y are defined by

$$IC_i(y) = C(y) - C(y_{N-i}) \quad (3.4)$$

which gives the additional costs when adding product i to the production line. The i th element of the vector y_{N-i} has a zero component, whereas all other components equal to those of y . It is clear that the average incremental costs are then given by:

³⁵ See Baumol W.J., Panzar J.C., Willig R.D.: Contestable Markets and the Theory of Industry Structure (1988) for a more detailed explanation.

$$AIC_i(y) = \frac{IC_i(y)}{y_i} \quad (3.5)$$

One can now define the degree of scale economies specific to any product i at an output vector y , holding the outputs of all other products constant:

$$S_i(y) = \frac{IC_i(y)}{y_i \cdot C_i(y)} = \frac{AIC_i}{MC_i} \quad (3.6)$$

Returns to scale for product i can then be increasing ($S_i(y) > 1$), decreasing ($S_i(y) < 1$) or constant ($S_i(y) = 1$).

The calculation of product-specific returns to scale for a subset $T \subseteq N$ is also possible. Here the output varies for two or more products, whereas all other output levels are held constant. Analogous to the approach above one must first define the incremental costs of the product set T by $IC_T(y) = C(y) - C(y_{N-T})$. The vector y_{N-T} comprises zero components for the products included in T whereas all other components equal to those of y . The average incremental costs of the product set T at y are given by $AIC_T = \frac{IC_T(y)}{\sum_{j \in T} y_j}$.

Analogous, a measurement of the degree of scale economies specific to the product set T is also feasible:

$$S_T = \frac{IC_T(y)}{\sum_{j \in T} y_j \cdot C_j(y)} = \frac{AIC_T}{MC_T} \quad (3.7)$$

The interpretation remains the same as above, i.e. there are increasing ($S_T > 1$), decreasing ($S_T < 1$) or constant returns to scale ($S_T = 1$) for the subset T . If $T = N$, this expression is identical to the multiproduct measure at the beginning of this chapter and if $T = \{i\}$ it is identical to the definition of the degree of scale economies specific to any product i .

3.3 Economies of Scope

Due to the fact that hospitals are multiproduct firms, it is likely that cost savings result only from the production of several different outputs in one hospital instead of generating each product by a specialized firm. This means that for a hospital it is possible to reduce its costs only by expanding the scope of its operations. The concept of jointness is closely related to those of economies of scope³⁶. If non-jointness in the production process occurs, the cost function can be written as:

$$C(y_1, \dots, y_n; w_1, \dots, w_m) = C^{(1)}(y_1; w_1, \dots, w_m) + \dots + C^{(n)}(y_n; w_1, \dots, w_m) \quad (3.8)$$

Economies of scope can be defined as follows:

³⁶ Also the concept of subadditivity is closely related to the one of economies of scope.

Let $P = \{T_1, \dots, T_k\}$ denote a nontrivial partition of $S \subseteq N$. That is, $\cup_i T_i = S$, $T_i \cap T_j = \emptyset$ for $i \neq j$, $T_i \neq \emptyset$, and $k > 1$. There are economies of scope at y_s with respect to the partition P if

$$\sum_{i=1}^k C(y_{T_i}) > C(y_s) \quad (3.9)$$

If no economies of scope are present, the above defined cost function (2.8) for non-jointness occurs. Additionally, weak economies of scope are present if the inequality under (3.9) is weak rather than strict and diseconomies of scope are present if the inequality sign is reversed. For the two-output case it follows that $N = \{1, 2\}$, $P = \{\{1\}, \{2\}\}$ and economies of scope exist if $C(y_1, y_2) < C(0, y_2) + C(y_1, 0)$.

After defining economies of scope in general, a formula to identify the degree of economies of scope for two subsets, T and $N - T$, is given:

$$SC_T(y) = \frac{C(y_T) + C(y_{N-T}) - C(y)}{C(y)} \quad (3.10)$$

This expression measures the relative increase in costs, which would result from producing the products comprised in the subset T and the products included in the subset $N - T$ separately. Adding the two subsets T and $N - T$ together results in the whole production set N . Such a fragmentation will increase the total costs of a firm ($SC_T(y) > 0$), decrease them ($SC_T(y) < 0$) or leave them unchanged ($SC_T(y) = 0$).

Economies of scale and economies of scope are linked by the following expression:

$$S_N(y) = \frac{\alpha_T S_T(y) + (1 - \alpha_T) S_{N-T}(y)}{1 - SC_T(y)} \text{ where } \alpha_T = \frac{\sum_{j \in T} y_j C_j}{\sum_{j \in N} y_j C_j} \quad (3.11)$$

If $SC_T(y) = 0$ (i.e. absence of any economies or diseconomies of scope), the overall economies of scale are only a weighted sum of product-specific economies of scale. If economies of scope are present ($SC_T(y) > 0$), the denominator is less than 1. This means that even if the product-specific economies of scale are constant, there will be increasing returns to scale over the entire product line. Moreover, if the product-specific economies of scale are decreasing, strong economies of scope can leave to overall returns to scale.

One can also identify economies of scope via the concept of cost complementarities. A twice-differentiable cost function exhibits weak cost complementarities over the product set N , up to y , if

$$\frac{\partial^2 C(\hat{y})}{\partial y_i \partial y_j} = C_{ij}(\hat{y}) \leq 0, \quad i \neq j, \text{ for all } \hat{y} \text{ with } 0 \leq \hat{y} \leq y \quad (3.12)$$

applies. As a consequence of this, the marginal costs of producing any particular product decrease weakly if the quantities of all other products increase. Economies of scope at y with respect to all partitions of N is then given, if a twice-differentiable multiproduct cost function exhibits weak cost complementarities over N , up to y .

Thus cost complementarities are a sufficient condition for economies of scope whereas the reverse does not apply.

If product-specific fixed costs are involved, the multiproduct cost function can be written as $C(y) = F(S) + c(y)$ with $C(y)$ being non differentiable along the relevant axes and $S = \{i \in N \mid y_i > 0\}$. A more confirmed proposition than the one mentioned before (3.12) is the following: If $c(y)$ is a twice-differentiable function, which exhibits weak cost complementarities over N , up to y , and if furthermore F is not superadditive (i.e. $F(S) + F(T) \geq F(S \cup T) \forall S, T \subseteq N$), then economies of scope at $y > 0$ with respect to all partitions of N occur.

3.4 Conclusion

Multiproduct economies of scale and scope are important concepts for multiproduct cost functions. Hence they play a significant role in merger decisions. The mere possibility of the exhibition of economies of scope represents an incentive for specialized firms to merge and to become multiproduct firms. Moreover, the possibility of extracting economies of scale or a potential expansion of already existing scale economies due to a merger will also be included into merger considerations.

As aforementioned, hospitals are special multiproduct firms because measuring the units of their outputs is – at first sight – a rather complex matter. The next chapter will deal with this problem.

4 Hospital Outputs and their Units of Measurement

Hospital outputs are the most important variables to include into a hospital cost function. Thus I will focus on the questions what the outputs of a hospital are and how they can be measured. For the sake of completeness, chapter 5 will show which other variables shall be included into a hospital cost function, so that in chapter 6 an analysis of hospital mergers using an appropriate hospital cost function can be given.

It is often difficult to measure different outputs from a service sector in general and hospitals in particular. Patients who want to improve their health status represent an output which is really hard to measure. A somewhat easier concept to quantify the output is to measure the treatment a hospital provides. For treatments hospitals are forced to combine different inputs (e.g.: diagnostic tests, nursing services, drugs, meals) to provide a diagnosis to cure the patient's illness or at least medicate its symptoms. As can be seen in chapter 5, it is also necessary to include these different inputs into the hospital cost function.

One can distinguish between four broad categories of hospital outputs:

- **Inpatient Treatment**
Patients who have to stay in the hospital while they obtain a treatment
- **Outpatient Treatment**
A treatment for persons who are not staying in the hospital
- **Teaching**
The existence of a doctor and/or nurse education
- **Research**
Expanding the stock of knowledge in medicine

In economics literature, authors mostly focus on inpatient treatment since this output category is the largest and also most important one. In addition to inpatient treatment also the outpatient treatment is often included into hospital cost functions, to allow for a more realistic approximation of hospital costs.

The teaching status can be included into the hospital cost function through a dummy variable. Nevertheless, Butler³⁷ and also Cowing & Holtmann³⁸ note that including the teaching status is not important when estimating a hospital cost function.

During my literature study I did not find any evidence for research as an important factor being included in a hospital cost function. Hence I assume that research is not relevant for the adopted hospital cost function in chapter 6. Moreover, research is mostly done in large university hospitals which are rarely involved into mergers.

³⁷ Butler J.R.G.: Hospital Cost Analysis (1995)

³⁸ Cowing T.G., Holtmann A.G.: Multiproduct Short-Run Hospital Cost Functions: Empirical Evidence and Policy Implications from Cross-Section Data (1983)

The treatment concept represents a given time period of hospitalization. Over this period the volume of output produced can be measured through two possible indicators: The number of patients discharged or the number of cases treated.

4.1 The Unit of Measurement – Cases vs. Days

Between authors who estimated hospital cost functions no consensus has been reached whether cases or days shall be used as the unit of measurement for the outputs. Lave & Lave³⁹ and Cowing & Holtmann⁴⁰ assumed the number of patient days as being representative whereas Butler⁴¹ and Schreyögg⁴² used the number of cases treated as their appropriate measure. Breyer⁴³ and Evans⁴⁴ used both indicators for their analysis.

By using a patient day as the unit of output measurement, the total output can be calculated through the number of patient days over a given time period. Nevertheless, the number of cases treated is a more defensible unit of output, since patient days could also be seen as inputs to produce a treated case. Furthermore, the number of patient days can differ enormously between hospitals which dismiss their patients on Fridays and readmit them on Mondays and others which do not follow this procedure. Moreover, hospitals trace different strategies how they treat their patients: More intensively over a shorter time period or less intensively over a longer time span. In both cases the same quantity of output will be produced while the number of patient days will vary significantly.

The connection between average costs per case (ACC) and average costs per day (ACD) can be given as:

$$TC_i = ACC_i \cdot n_i = ACD_i \cdot d_i \quad (4.1)$$

The total costs for the i^{th} hospital is given by its average costs per case times the number of cases treated, which equals its average costs per day times the total number of patient days. This expression can be rewritten as

$$ACC_i = ACD_i \cdot ALS_i \text{ where } ALS_i = \frac{d_i}{n_i} \quad (4.2)$$

The average costs per case for hospital i are given by the average costs per day times the average length of stay. The relationship (4.2) can also be shown graphically:

³⁹ Lave J.R., Lave L.B.: Hospital Cost Functions (1970)

⁴⁰ Cowing T.G., Holtmann A.G.: Multiproduct Short-Run Hospital Cost Functions: Empirical Evidence and Policy Implications from Cross-Section Data (1983)

⁴¹ Butler J.R.G.: Hospital Cost Analysis (1995)

⁴² Schreyögg J.: A Micro-Costing Approach to Estimating Hospital Costs for Appendectomy in a Cross-European Context (2008)

⁴³ Breyer F.: The Specification of a Hospital Cost Function: A Comment on the Recent Literature (1987)

⁴⁴ Evans R.G.: "Behavioural" Cost Functions for Hospitals (1971)

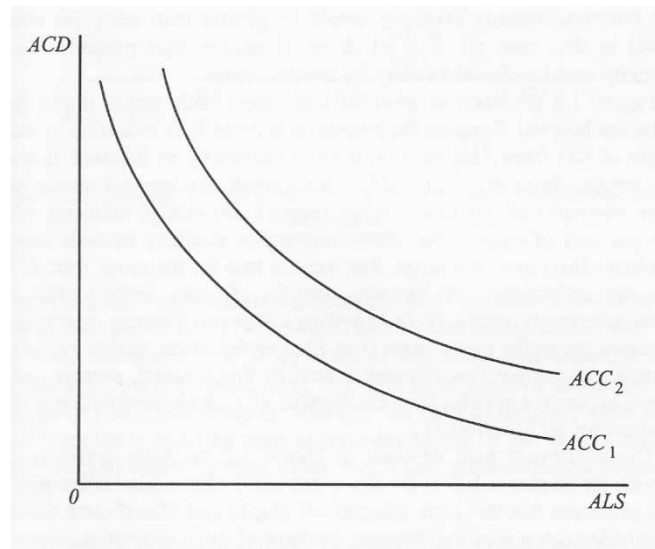


Figure 3 Average Costs per Day, Average Length of Stay and Constant Average Costs per Case Contours
Butler J.R.G. (1995)

Figure 3 shows that the longer the average length of stay the less the average costs per day. This can also be seen in the next figure where a hypothetical patient cost profile is given:

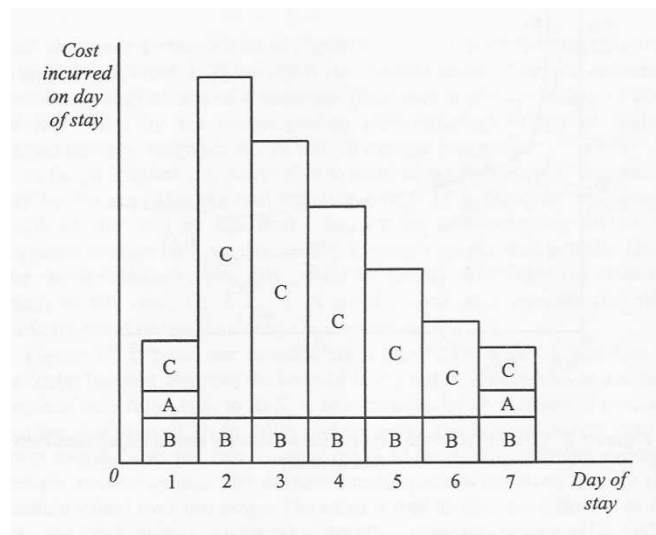


Figure 4 A hypothetical Patient Cost Profile
Butler J.R.G. (1995)

A represents the fixed costs of admission and discharge, B equals the hotel costs and C are the treatment costs. The length of stay s is 7 days. Figure 4 illustrates that the treatment costs are highest at the second day of hospitalization and decline afterwards. It can be assumed that the treatment costs in figure 4 are relatively small at the first day when often only the admission is arranged.

The total costs of patient i in hospital j can be derived when adding the fixed costs of admission and discharge, the hotel costs, and the treatment costs for every day the patient is staying in the hospital:

$$TC_{ij} = \sum_{k=1}^s A_{ijk} + B_{ijk} + C_{ijk} \quad (4.3)$$

The average costs per day are then given by

$$\frac{TC_{ij}}{s} \quad (4.4)$$

where s denotes the length of stay.

Returning to figure 3, one will note that the value of the intensity elasticity along the average costs per case contour equals -1 . This means that the proportionate changes in average length of stay and average costs per day are equal but grow in opposite directions (i.e. if average length of stay decreases then average costs per day increase by the same amount). In reality hospitals are not able to move along a given ACC contour, which means that actually $E \gtrsim |1|^{45}$.

The duration/intensity elasticity of average length of stay with respect to average costs per day is given by:

$$E = \frac{\Delta ALS / ALS}{\Delta ACD / ACD} \quad (4.5)$$

One can conclude that a comparison between hospitals – especially in case of productive efficiency – is very difficult if the number of patient days is the chosen output unit. If the average length of stay decreases, the average costs per day increase. Different average costs per day result only through variation of the length of stay. This fact supports the view that the number of cases treated shall be the selected output unit. The unsuitability of the number of patient days as dependable unit for the measurement of outputs causes conflicting results in various papers for the investigation of economies of scale: Carr & Feldstein⁴⁶ find in their analysis of 3147 non-profit general hospitals in the U.S. that economies of scale in the hospital market occurs up to a given output level. In contrast Ingbar & Taylor⁴⁷ find an inverted U-shaped average cost curve for 72 non-profit hospitals in Massachusetts, which indicates, that up to a given output level (in their analysis < 200 beds) diseconomies of scale are present. Moreover, Carr & Feldstein⁴⁸ estimate a correlation between costs per day and costs per case of only 0,232.

One reason why the number of patient days is used in several studies as the unit of output is its simplicity. A treated case on the other hand is not a homogeneous unit of output, not even within one single hospital, since different diseases require for different quantities and types of inputs (e.g. medication, nursing service, use of

⁴⁵ Butler J.R.G.: Hospital Cost Analysis (1995)

⁴⁶ Carr W.J., Feldstein P.J.: The Relationship of Cost to Hospital Size (1967)

⁴⁷ Ingbar M.L., Taylor L.D.: Hospital Costs in Massachusetts (1968)

⁴⁸ Carr W.J., Feldstein P.J.: The Relationship of Cost to Hospital Size (1967)

medical technology equipment, etc.). This results in the necessity to introduce the concept of case mix: When estimating hospital cost functions, specific data assigned to specific types of illnesses has to be available to take the case mix into account. Since these hospital data are not always easily available, the number of patient days is often the only left output choice to estimate these functions.

4.2 Case Mix

As mentioned above, the number of cases treated is the preferable unit of output, but it is obvious that not all treatments belong to the same class of output. Treating a patient with e.g. tonsillitis is less intensive and costly than treating a patient who is afflicted with cancer. Thus it is important to include variables for the case mix in the regression analysis for estimating a hospital cost function. These variables account for the mix of cases treated by a hospital and hence describe how these different groups of illnesses influence the costs of the hospital. It has to be noted, that the case mix varies between, but not within hospitals (i.e. every hospital treats different cases). Furthermore, the selected case mix is fixed over a given estimation period. Two possibilities exist to distinguish between different outputs in the hospital cost function: The case mix classification scheme and the case mix index.

4.2.1 The Case Mix Classification Scheme

The two most widely used case mix classification schemes are the international classification of diseases (*ICD*) and the diagnosis-related groups (*DRG*).

The so called *ICD* codes are published by the World Health Organization and represent the most detailed disease classification available. The *ICD* codes refer to a set of possible output categories of a hospital, containing 17 major chapter headings. These major headings are fragmented into around 1000 categories of 3-digit codes and into more than 1000 categories of 4-digit codes. Currently the 9th revision of the codes (*ICD-9*) of the international statistical classification of diseases, injuries and causes of death is being published.

Another approach to cover the case mix classification scheme is the diagnosis-related groups which are “perhaps the most well known and widely applied case-mix measures”⁴⁹. The diagnosis-related groups define different case types, whereas each of these types is expected to receive similar outputs or services from a hospital. The original version of the *DRGs* included 83 major diagnostic categories (*MDCs*) based on primary diagnosis. These *MDCs* were subdivided into more homogeneous groups resulting in 383 patient classes. There is also a revised and medically more meaningful version of the *DRGs* present. It comprises 23 major diagnostic groups, mostly defined in terms of the organ system affected. At the moment 467 *DRGs* and also 3 additional patient classes – for patients who had surgical procedures which were unrelated to their principal diagnosis – exist.

The primary problem of the *ICD* codes and also *DRGs* is that within every group, heterogeneity of cases treated is present. It is also obvious that both methods

⁴⁹ Hornbrook M.C.: Hospital Case Mix: Its Definition, Measurement and Use: Part II (1982)

contain too many output categories for statistical estimations of hospital cost functions. For that reason the concept of the scalar case mix indices will be explained.

4.2.2 The Scalar Case Mix Indices

A scalar case mix index obtains the possibility for a single-valued measurement of the output composition of a hospital. Because of the multiproduct nature of the hospital, an index through the use of weights incorporated into an output aggregator function (= a linear or non-linear weighted sum of individual outputs) has to be constructed. The weights are used to reflect the heterogeneity between different case mix categories.

Hornbrook⁵⁰ notices, that such an index consists of three components: A case mix classification scheme (see section 4.2.1), a weighting scheme which is necessary to establish relativities between the different case types to allow for meaningful aggregation, and a linear or non-linear aggregation formula.

The most well known scalar case mix index is the one introduced by Evans & Walker⁵¹ and is named "The Information Theory Index". There the method is the following:

The proportion of the cases of the i^{th} hospital which are falling into the j^{th} diagnostic category is given by $p_{ij} = \frac{n_{ij}}{N_i}$ and the proportion of cases of the j^{th} type which are treated in the i^{th} hospital are given by $q_{ij} = \frac{n_{ij}}{N_j}$. There are also several conditions implemented: $\sum_j p_{ij} = \sum_i q_{ij} = 1$, $\sum_i p_{ij} \neq 1$, $\sum_j q_{ij} \neq 1$.

The proportion of all cases treated in the i^{th} hospital is defined as $p_i = \frac{N_i}{N}$ and the proportion that all cases are falling in the j^{th} category is given by $q_j = \frac{N_j}{N}$.

The corresponding matrix with the hospital case mix data is shown in table 1 (p.33).

Moreover, Evans & Walker defined two information measures for the j^{th} case type:

$$H_j^1 = \sum_i q_{ij} \ln \left[\frac{q_{ij}}{\frac{1}{I}} \right] \quad (4.6)$$

$$H_j^2 = \sum_i q_{ij} \ln \left[\frac{q_{ij}}{p_i} \right] \quad (4.7)$$

The logarithmic term shows the information gain from learning that the event $\frac{1}{I}$ (i.e. all hospitals treat an equal proportion of case type j) or p_i respectively (i.e. all cases of type j are treated in the i^{th} hospital) took place.

The term $\sum_i q_{ij}$ represents the probability that case type j is treated in hospital i . The information gain in the first formula (4.6) is equal to zero due to the same case

⁵⁰ Hornbrook M.C.: Hospital Case Mix: Its Definition, Measurement and Use: Part II (1982)

⁵¹ Evans R.D., Walker H.D.: Information Theory and the Analysis of Hospital Cost Structure (1972)

distribution among all hospitals. This can be displayed by substituting $q_{ij} = \frac{1}{I}$ into equation (4.6), which results for the logarithmic term and thus for the information gain in $\ln(1) = 0$. The more distinct the distribution of cases among hospitals, the higher the information gain.

	Diagnostic category								
	1	2	.	.	j	.	.	J	
1	n_{11}	n_{12}	.	.	n_{1j}	.	.	n_{1J}	N_1
2	n_{21}	n_{22}	.	.	n_{2j}	.	.	n_{2J}	N_2
.
.
.
i	n_{i1}	n_{i2}	.	.	n_{ij}	.	.	n_{iJ}	N_i
.
.
.
I	n_{I1}	n_{I2}	.	.	n_{IJ}	.	.	n_{IJ}	N_I
	$N_{.1}$	$N_{.2}$.	.	$N_{.j}$.	.	$N_{.J}$	N

n^{ij} = number of cases treated in the i^{th} hospital in the j^{th} diagnostic category
 $N_i = \sum_{j=1}^J n_{ij}$ = total number of cases treated in the i^{th} hospital
 $N_j = \sum_{i=1}^I n_{ij}$ = total number of cases treated in the j^{th} diagnostic category
 $N = \sum_{i=1}^I N_i = \sum_{j=1}^J N_j$ = total number of cases treated in all hospitals in all diagnostic categories

Table 1 Hospital Case Mix Data
Butler J.R.G. (1995)

In the next step the authors standardize the two information measures (4.6 and 4.7) to obtain a mean of unity:

$$\overline{H_j^1} = \frac{H_j^1}{\sum_j H_j^1 q_j} \quad (4.8)$$

$$\overline{H_j^2} = \frac{H_j^2}{\sum_j H_j^2 q_j} \quad (4.9)$$

Moreover, $\overline{H_j^1}$ and $\overline{H_j^2}$ are used as weights in a linear aggregation formula to produce two indices which represent the relative complexity of a hospitals case load:

$$X_i^1 = \sum_j \overline{H_j^1} p_{ij} \quad (4.10)$$

$$X_i^2 = \sum_j \overline{H_j^2} p_{ij} \quad (4.11)$$

These two measurements (4.10 and 4.11) – i.e. weighted sums of case mix proportions to measure the various outputs – can now be used for estimating hospital cost functions.

The major advantage of case mix indices is that, unlike the *ICD* codes or *DRGs* they reduce the dimensionality of the data. Furthermore, these indices can be applied to any case mix classification scheme. Nevertheless, there is also a major disadvantage: It is possible that the index shows identical values for hospitals with various underlying case mixes.

4.3 Conclusion

This chapter deals with the problem which unit of output shall be chosen together with the question how different hospital outputs can be measured. The decision which unit of output shall be preferred is relatively straight sailing: Since patient days differ enormously between hospitals, the number of cases treated is a more robust variable, especially for comparison of different hospital costs with each other. The flaw is that not all cases treated are equal. Consequently, the concept of case mix has to be applied, and hence detailed information about different diseases and their treatment is required. Estimating hospital cost functions, the case mix can be represented via the case mix classification schemes or the scalar case mix indices. The use of the case mix classification scheme seems to be simpler but requires a large number of explanatory variables. However, when using the scalar case mix indices, the number of explanatory variables will be noticeably reduced while it is possible that identical values for hospitals with different outputs treated occur.

After having specified the output unit to be used and having discussed the possible approaches to measure hospital outputs, I want to show in the next chapter, that different authors deal with different units of outputs when estimating hospital cost functions. In addition, they also use unequal variables to explain the variation in hospital costs. Before analyzing the effects of a horizontal merger, I will thus investigate which variables are necessary to be included into these cost functions.

5 Hospital Cost Functions

Numerous authors⁵² have estimated hospital cost functions in order to specify the factors that influence costs. While different approaches have been implemented (as discussed in chapter 4) no consensus has been reached on the preferred unit of output to be taken.

Some authors use the number of patient days as the dependent variable whereas others prefer the number of cases treated. As noted above, this difference results from the availability of data. Nevertheless, the choice of variables used to explain hospital costs is a question at issue. In this chapter I will describe the hospital cost function according to Butler⁵³, which differs from the hospital cost functions estimated by Cowing & Holtmann⁵⁴, Sinay & Campbell⁵⁵, or Wang & Zhao & Mahmood⁵⁶ respectively. Although the hospital cost functions estimated by the last-mentioned authors are very similar, the regression outputs provide different results which variables are appropriate for the explanation of variations in hospital costs. After having identified the relevant explanatory variables for a hospital cost function by comparing the different statistical significances of the estimated coefficients, a hospital cost function for the analysis will be assumed in chapter 6.

Breyer⁵⁷ identifies two approaches to formulate hospital cost functions: The ad-hoc specification and the flexible functional form. The simplest form of the ad-hoc specification is the additive-linear one. In this case the vector of all explanatory variables of the h^{th} hospital can be described as $x_h = (x_{ih}, \dots, x_{nh})$ with, for instance, the bed size of the hospital (to specify the capacity), the case mix, the wage level of hospital employees, dummy variables for the teaching status (a measurement of quality), the average length of stay or the case flow rate (the number of cases treated per bed per year) being included. The cost per case or per patient day for hospital h is then given by

$$C_h = \alpha_0 + \sum_{i=1}^n \alpha_i x_{ih} + u_h \quad (5.1)$$

with α_i being the structural parameters to be estimated. The ad-hoc specification is restricted by the different linear and additive separable cost determinants. That means an additional day of care raises the costs by a fixed amount without any consideration of the hospital's capacity and utilization (i.e. constant returns to scale are assumed).

The second approach introduced by Breyer to estimate a hospital cost function is via a flexible functional form derived from microeconomic theory. It indicates that the cost function represents the minimum costs for the production of a given volume of output

⁵² E.g.: Lave J.R., Lave L.B.: Hospital Cost Functions (1970), Butler J.R.G.: Hospital Cost Analysis (1995), Wang J., Zhao Z., Mahmood A.: Relative Efficiency, Scale Effect, and Scope Effect of Public Hospitals: Evidence from Australia (2006)

⁵³ Butler J.R.G.: Hospital Cost Analysis (1995)

⁵⁴ Cowing T.G., Holtmann A.G.: Multiproduct Short-Run Hospital Cost Functions: Empirical Evidence and Policy Implications from Cross-Section Data (1983)

⁵⁵ Sinay U., Campbell C.: Scope and Scale Economies in Merging Hospitals Prior to Merger (1995)

⁵⁶ Wang J., Zhao Z., Mahmood A.: Relative Efficiency, Scale Effect, and Scope Effect of Public Hospitals: Evidence from Australia (2006)

⁵⁷ Breyer F.: The Specification of a Hospital Cost Function: A Comment on the Recent Literature (1987)

as a function of exogenous input prices. The explanatory variables contain only output quantities and input prices. In addition, the neoclassical production theory suggests that this cost function fulfills all the properties mentioned at the beginning of chapter 2. The general formula of a flexible functional form is given by

$$g(C) = \alpha_0 + \sum_{i=1}^m \alpha_i f_i(x_i) + \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^n \beta_{ij} f_i(x_i) f_j(x_j) \quad (5.2)$$

$$\beta_{ij} = \beta_{ji} \quad \forall i, j$$

where C represents the total costs and (x_1, \dots, x_n) are the explanatory variables. Breyer also noted that the most commonly used form of the general formula is the translog (i.e. transcendental logarithmic) one. In this case $g(C) = \ln C$ and $f_i(x_i) = \ln x_i$. A flaw in the use of the translog cost function is the fact that a zero value for any explanatory variable is inadmissible since the logarithm of 0 is undefined. This means that all hospitals have to produce positive amounts of all outputs. On the other hand it is possible to test for economies of scale by incorporating appropriate parametric restrictions⁵⁸ when using this function. The simplest representation of a flexible functional form is given by a quadratic formula where $g(C) = C$ and $f_i(x_i) = x_i$.

For all flexible functional forms another restriction has to be taken into account. Due to the numerous variables needed an accurate reflection of the patient's heterogeneity (via case mix) is difficult to handle in a flexible functional form model (e.g.: using a translog cost function $[(m+n) \cdot (m+n+1)/2]$ parameters have to be estimated for m outputs and n inputs⁵⁹).

In economics literature, several approaches for the estimation of hospital cost functions have been published. In the following section, the different estimation procedures in general and the specific variables included in particular shall be evaluated. This investigation will provide the base for the accomplishment of the merger analysis in chapter 6 of this thesis.

5.1 Estimated Hospital Cost Functions

One can distinguish between short-run and long-run cost functions. Generally, in the short-run some input factors are fixed whereas in the long-run all inputs are variable. Butler⁶⁰ estimates both versions using the number of beds (B) to capture the size of a hospital. A hospital's capacity to accommodate patients is given by the rated bed days:

$$RBD = 365 B \quad (5.3)$$

Due to the fact that hospital beds are normally not homogeneous and thus not perfectly substitutable, this approach must be regarded as simplification (e.g.: a bed at the intensive care is connected with a higher treatment than a bed at the maternity ward).

⁵⁸ See Butler J.R.G.: Hospital Cost Analysis (1995)

⁵⁹ See Brown R.S., Caves D.W., Christensen L.R.: Modelling the Structure of Cost and Production for Multiproduct Firms (1979)

⁶⁰ Butler J.R.G.: Hospital Cost Analysis (1995)

Butler also uses the case flow rate, defined as the number of cases treated per bed per year as an explanatory variable to define the average costs per case:

$$CFR = \frac{y}{B} \quad (5.4)$$

The occupancy rate, which is also used in his regression analysis, can be described by the occupied bed days (*OBD* – a hospital's total number of occupied bed days per year) and the rated bed days (see 5.3):

$$OCC = \frac{OBD}{RBD} \quad (5.5)$$

The case flow rate and the occupancy rate are linked by the following relationship:

$$CFR = \frac{OCC}{ALS} \cdot 365 \quad (5.6)$$

where *ALS* stands for average length of stay and is given by:

$$ALS = \frac{OBD}{y} \quad (5.7)$$

The case flow rate has a maximum value of 365 which represents an occupancy rate of one (i.e. 100 %), and the average length of stay being its minimum value of one day. An increase in the number of cases treated per bed per year (= *CFR*) can be caused by an increase in the occupancy rate and/or a reduction in the average length of stay. These possible changes induce also differences between hospitals in marginal costs of treating an additional patient.

5.1.1 Ad-hoc Specifications

For his empirical studies Butler⁶¹ uses a linear total cost function which refers to the group of ad-hoc specifications (described at the beginning of chapter 5). This function can be formulated as follows:

$$C = \sum_i a_i y_i \quad (5.8)$$

a_i denotes the average and marginal costs of product i , and y_i denotes the number of cases treated in the i^{th} diagnostic category. The total output y is given by:

$$y = \sum_i y_i \quad (5.9)$$

Due to the arguments stated in section 4.1, the author uses the average costs per case as the unit of output:

$$ACC = \frac{C}{y} = \sum_i a_i p_i \quad (5.10)$$

The average costs per case correspond to the ray average costs described in section 3.2 and

⁶¹ Butler J.R.G.: Hospital Cost Analysis (1995)

$$p_i = \frac{y_i}{\sum_i y_i} \quad (5.11)$$

measures the hospital's proportion in each diagnostic category.

If p_i does not change as the total volume of cases treated differ, then the average costs per case remain constant, meaning that there are overall constant returns to scale.

To allow for scale effects, Butler includes rated bed days and rated bed days squared into the above defined cost function (5.8):

$$C = \sum_{i=1}^n a_i y_i + r_1 RBD + r_2 RBD^2 \quad /: y \quad (5.12)$$

$$\begin{aligned} ACC &= \sum_{i=1}^n a_i p_i + r_1 \left(\frac{RBD}{y} \right) + r_2 \left(\frac{RBD^2}{y} \right) \\ &= \sum_{i=1}^n a_i p_i + r_1 \left(\frac{365}{CFR} \right) + r_2 \left(\frac{365 \cdot RBD}{CFR} \right) \end{aligned} \quad (5.13)$$

In order to avoid the problem that a reduction in the average length of stay increases the number of cases treated per bed per year⁶² (see 5.6 for the relationship), Butler also includes average length of stay into the equation. Furthermore, the author includes the number of beds in linear and quadratic terms as independent variables to possibly obtain a conventional U-shaped cost curve:

$$ACC = \sum_{i=1}^n a_i p_i + r_1 \left(\frac{365}{CFR} \right) + r_2 \left(\frac{365 \cdot RBD}{CFR} \right) + r_3 ALS + r_4 B + r_5 B^2 \quad (5.14)$$

Schuttinga⁶³ recommends, that the average length of stay shall not be included in an average cost function because an additional day spend in a hospital also depends on the price charged. Consequently, the average length of stay is not an exogeneous variable.

For one of his numerous empirical analyses Butler uses data from 121 public hospitals in Queensland (Australia) for the years 1977 – 1978 together with 17 and 47 diagnostic categories respectively to account for the case mix.

Butler observes that size as the only explanatory variable is very poor for explaining average costs ($\overline{R^2} = 0,06$). When estimating equation (5.14) the parameters of B and B^2 are statistically significant which means that economies of scale are present (see table 2⁶⁴). The regression output also indicates that using 47 diagnostic categories (DCs) explains the variation in the average costs per case better than using 17 categories. Thus, one can conclude that the more aggregated the case mix classification scheme, the less is its explanatory power.

⁶² Which certainly results in different costs.

⁶³ See Butler J.R.G.: Hospital Cost Analysis (1995)

⁶⁴ The first term of (5.14) is not shown in the regression output.

	$\frac{365}{CFR}$	$\frac{365RBD}{CFR}$	ALS	B	B ²	\bar{R}^2	SEE	F
18DCs+(b) (d.f.=88)	3.115 (3.67*)	0.00027 (3.18*)	14.459 (2.53**)	-2.035 (-3.43*)	0.00065 (2.41**)	.69	133.72	9.22*
47DCs+(b) (d.f.=59)	4.136 (3.85*)	0.00031 (3.28*)	17.368 (2.42**)	-2.634 (-3.56*)	0.00092 (2.58**)	.77	115.23	7.49*

Notes: * Significant at 1% level.
 ** Significant at 5% level.
 (a) t-values in parentheses.
 (b) The "+" sign indicates that the additional case mix dimensions have been added to the specification.
 d.f. = degrees of freedom.

Source: Regression results.

Table 2 Regression Output
Butler J.R.G. (1995)

Butler also finds a U-shaped average cost curve for the long-run with the minimum point at 469 beds. Furthermore, the author states that a higher case flow rate demands a higher number of beds to obtain economies of scale. One should note that re-estimating equation (5.14) with other specifications weakens the evidence of scale effects and also the long-run U-shaped average cost curve.

In addition, Butler demonstrates that an increase in the case flow rate will reduce the average costs per case by a greater amount if it is caused by a reduction of the average length of stay rather than by an increase in the occupancy rate. Furthermore, a reduction in the average costs per case is larger for smaller initial case flow rates.

Input price differences are one possible reason for cost variations between hospitals (e.g.: regional differences in the wage rates). Butler suggests that the variation in average costs per case due to different input prices is minimal and therefore not included in his analysis.

He also compares costs of teaching and non-teaching public hospitals in Queensland. When accounting for teaching in the cost function it is important to review the case mix variations because teaching hospitals treat a more expensive and complex case mix. Teaching hospitals may also generate higher costs since they are confronted with indirect teaching costs due to the learning process and the necessity to provide the newest equipment for academic purpose. Moreover, teaching hospitals are generally larger in size and thus treat a higher number of inpatients in comparison with non-teaching hospitals.

The most common approach to identify teaching and non-teaching hospitals is to include dummy variables in the regression equation. After including the teaching dummy variables in his analysis and adjusting for the case mix and all other relevant factors, Butler concludes that the teaching status has virtually no impact on the hospital costs. He states that most of the arising expenses are common costs and not separable teaching costs.

In addition to the comparison of teaching and non-teaching hospitals, differences in costs between public and private hospitals have to be taken into account for further investigations. Butler uses data from 120 public and 38 private⁶⁵ hospitals in Queensland (non-profit and profit which are regulated by the government) for the years 1977 – 1978. He finds that though private hospitals are smaller in size compared to public ones, they discharge the same number of patients. Private hospitals also show a shorter average length of stay and a higher occupancy rate. This results in a higher case flow rate and in turn implies, that private hospitals have lower average costs per case. There are also significant differences between public and private hospitals with respect to the case types treated. Butler states that it is common practice for profit-orientated hospitals to choose case types with relatively high price-cost margins and leave all other case types to be treated by non-profit or public hospitals.

Having estimated equations with different specifications he arrives at the conclusion that the average costs per case are substantially lower in private hospitals due to the differences in the case mix treated.

As a matter of course Butler is not the only author who estimates hospital cost functions⁶⁶. The majority of these authors do not choose Butler's approach of an ad-hoc specification, but estimate a flexible functional form. As mentioned at the beginning of this chapter, the most widely used specification of flexible functional forms is the translog cost function.

5.1.2 Flexible Functional Forms - Translog Cost Functions

Cowing & Holtmann⁶⁷ estimate a multiproduct translog variable cost function with data from 138 New York State general-care hospitals (non-profit and profit) for the year 1975. They use patient days as their unit of output.

The short-run variable cost function which minimizes the variable hospital costs can be written as: $C^V = G(Y, p', K, A)$ where Y is the vector of outputs, p' is the vector of variable input prices, K is the vector of fixed capital inputs, and A represents the fixed admitting physician inputs.

The long-run costs are then given by: $C = C^V(Y, p', K, A) + p_K K + p_A A$ where p_K specifies the user costs of capital and p_A describes the price of admitting physician services. The last two terms of the cost function represent the fixed costs. Since the short-run variable cost function has already been minimized, the envelope conditions for long-run cost minimizing behavior are given as follows:

$$\frac{\partial C^V}{\partial K} = -p_K \quad (5.15)$$

$$\frac{\partial C^V}{\partial A} = -p_A \quad (5.16)$$

⁶⁵ Specific data for 3 of 38 private hospitals available; data for 35 hospitals aggregated.

⁶⁶ E.g.: Cowing T.G., Holtmann A.G.: Multiproduct Short-Run Hospital Cost Functions: Empirical Evidence and Policy Implications from Cross-Section Data (1983), Sinay U., Campbell C.: Scope and Scale Economies in Merging Hospitals Prior to Merger (1995), Lave J.R., Lave L.B.: Hospital Cost Functions (1970)

⁶⁷ Cowing T.G., Holtmann A.G.: Multiproduct Short-Run Hospital Cost Functions: Empirical Evidence and Policy Implications from Cross-Section Data (1983)

This means that if the result of (5.15) is greater than $-p_K$ over-investment in capacity and equipment occurs.

Cowing & Holtmann take the following short-run (using the quantity of capital as a fixed input) translog variable cost function as basis for their estimations:

$$\begin{aligned} \ln C^V = & \alpha_0 + \sum_r \alpha_r \ln Y_r + \frac{1}{2} \sum_r \sum_s \alpha_{rs} \ln Y_r \ln Y_s + \sum_i \beta_i \ln p'_i \\ & + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln p'_i \ln p'_j + \gamma_K \ln K + \gamma_{KK} (\ln K)^2 + \phi_A \ln A \\ & + \phi_{AA} (\ln A)^2 + \sum_r \sum_i \delta_{ri} \ln Y_r \ln p'_i + \sum_r \phi_r \ln Y_r \ln K \\ & + \sum_r \theta_r \ln Y_r \ln A + \sum_i \vartheta_i \ln p'_i \ln K + \sum_i \pi_i \ln p'_i \ln A \\ & + \gamma_{KA} \ln K \ln A \end{aligned} \quad (5.17)$$

Y represents a vector of 5 diagnostic categories: Emergency room care, medical-surgical care, pediatric care, maternity care and other care. p' contains 6 variable input prices for nursing labor, professional labor, administrative labor, general labor, materials, and supplies. K is a single measure of a fixed capital stock and A represents the number of fixed admitting physicians in each hospital. In addition, Cowing & Holtmann add dummy variables in their regression analysis to distinguish between teaching and non-teaching hospitals as well as between profit and non-profit hospitals.

In order to have a variable cost function being homogeneous of degree 1 (if doubling all input prices the total variable costs will double) Cowing & Holtmann restrict the following parameters:

$$\sum \beta_i = 1, \quad \sum \delta_{ri} = 0, \quad \sum \beta_{ij} = 0, \quad \sum \vartheta_i = 0, \quad \sum \pi_i = 0 \quad (5.18)$$

Cowing & Holtmann's estimation results of (5.17) are given below:

Variable	Parameter	Estimate	(t-value)
Constant	α_0	-.0467	(-0.93)
Medical-Surgical	α_1	.5796	(5.00)
Maternity	α_2	.0832	(2.57)
Pediatrics	α_3	-.0581	(-0.84)
Other Inpatient	α_4	.1262	(5.66)
Emerg. Room	α_5	.1272	(1.43)
(Med-Surg) ²	α_{11}	.0763	(0.33)
(Maternity) ²	α_{22}	.0139	(2.20)
(Pediatrics) ²	α_{33}	.0026	(0.48)
(Other Inp.) ²	α_{44}	.0212	(4.78)
(Emer. Room) ²	α_{55}	-.0218	(-0.52)
$MS \times MAT$	α_{12}	.0040	(0.31)
$MS \times PED$	α_{13}	-.0101	(-0.41)
$MS \times Other$	α_{14}	-.0196	(-1.60)
$MS \times EMR$	α_{15}	.0404	(0.51)
$MAT \times PED$	α_{23}	-.00004	(-0.05)
$MAT \times Other$	α_{24}	.0004	(0.52)
$MAT \times EMR$	α_{25}	.0029	(0.59)

Variable	Parameter	Estimate	(t-value)
$PED \times Other$	α_{34}	-.0076	(-1.08)
$PED \times EMR$	α_{35}	-.0041	(-0.74)
$Other \times EMR$	α_{45}	.0084	(0.91)
Nursing	β_1	.2224	(18.3)
Ancillary	β_2	.0566	(21.7)
Professional	β_3	.1612	(21.8)
Administrative	β_4	.0764	(28.3)
General	β_5	.1704	(16.0)
Drugs & Supplies	β_6	.3130	(40.3)
(Nursing) ²	β_{11}	.0084	(0.41)
(Ancil.) ²	β_{22}	.0537	(6.30)
(Prof.) ²	β_{33}	.0656	(5.85)
(Admin.) ²	β_{44}	.0243	(3.33)
(General) ²	β_{55}	.0893	(4.81)
(Drs. & Sup.) ²	β_{66}	.1347	(6.21)
Nurs. \times Ancil.	β_{12}	-.0063	(-1.23)
Nurs. \times Prof.	β_{13}	.0251	(2.58)
Nurs. \times Admin.	β_{14}	.0033	(0.64)
Nurs. \times Gen.	β_{15}	.0138	(0.88)
Nurs. \times DS	β_{16}	-.0444	(-3.09)
Ancil. \times Prof.	β_{23}	-.0052	(-1.01)
Ancil. \times Admin.	β_{24}	-.0133	(-2.37)
Ancil. \times Gen.	β_{25}	-.0233	(-3.68)
Ancil. \times DS	β_{26}	-.0057	(-0.76)
Prof. \times Admin.	β_{34}	.0065	(1.27)
Prof. \times Gen.	β_{35}	-.0517	(-4.83)
Prof. \times DS	β_{36}	-.0257	(-1.81)
Admin. \times Gen.	β_{45}	-.0024	(-0.37)
Admin. \times DS	β_{46}	-.0404	(-3.44)
Gen. \times DS	β_{56}	-.0185	(-2.59)
Capital	α_x	.1015	(1.98)
(Capital) ²	α_{xx}	.0501	(1.24)
Adm. Physicians	ϕ_A	.0973	(1.41)
(Adm. Phy.) ²	ϕ_{AA}	.0152	(0.20)
MS \times Nurs.	δ_{11}	.0034	(0.16)
MS \times Ancil.	δ_{12}	.0003	(0.05)
MS \times Prof.	δ_{13}	.0013	(0.11)
MS \times Admin.	δ_{14}	.0007	(0.15)
MS \times Gen.	δ_{15}	.0229	(1.26)
MS \times DS	δ_{16}	-.0285	(-2.09)
MAT \times Nurs.	δ_{21}	-.0009	(-0.66)
MAT \times Ancil.	δ_{22}	.0005	(1.53)
MAT \times Prof.	δ_{23}	.0003	(0.38)
MAT \times Admin.	δ_{24}	-.0001	(-0.29)
MAT \times Gen.	δ_{25}	.0008	(0.66)
MAT \times DS	δ_{26}	-.0006	(-0.64)
Ped. \times Nurs.	δ_{31}	.0028	(1.54)
Ped. \times Ancil.	δ_{32}	-.0003	(-0.80)
Ped. \times Prof.	δ_{33}	-.0005	(-0.43)
Ped. \times Admin.	δ_{34}	-.00002	(-0.05)
Ped. \times Gen.	δ_{35}	-.0010	(-0.63)
Ped. \times DS	δ_{36}	-.0010	(-0.84)
Other \times Nurs.	δ_{41}	-.00002	(-0.01)
Other \times Ancil.	δ_{42}	-.0013	(4.25)
Other \times Prof.	δ_{43}	.0011	(1.27)
Other \times Admin.	δ_{44}	.0007	(2.23)
Other \times Gen.	δ_{45}	-.00002	(-0.02)
Other \times DS	δ_{46}	-.0005	(-0.51)
EMR \times Nurs.	δ_{51}	-.0231	(-2.90)
EMR \times Ancil.	δ_{52}	.0082	(4.72)
EMR \times Prof.	δ_{53}	.0100	(2.04)
EMR \times Admin.	δ_{54}	.0007	(0.41)
EMR \times Gen.	δ_{55}	.0022	(0.31)
EMR \times DS	δ_{56}	.0021	(0.41)

Variable	Parameter	Estimate	(t-value)
$MS \times CAP$	ψ_1	-.0438	(-0.74)
$MAT \times CAP$	ψ_2	-.0063	(-1.41)
$PED \times CAP$	ψ_3	.0062	(0.93)
$Other \times CAP$	ψ_4	.0006	(0.10)
$EMR \times CAP$	ψ_5	.0006	(0.02)
$MS \times APH$	θ_1	.0321	(0.28)
$MAT \times APH$	θ_2	-.0084	(-1.22)
$PED \times APH$	θ_3	.0012	(0.10)
$Other \times APH$	θ_4	.0084	(1.12)
$EMR \times APH$	θ_5	-.0440	(-1.17)
$Nurs. \times CAP$	ξ_1	-.0039	(-0.48)
$Ancil. \times CAP$	ξ_2	-.0020	(-1.10)
$Prof. \times CAP$	ξ_3	.0045	(0.89)
$Admin. \times CAP$	ξ_4	-.0043	(2.32)
$Gen. \times CAP$	ξ_5	-.0072	(-1.00)
$DS \times CAP$	ξ_6	.0129	(2.43)
$Nurs. \times APH$	π_1	-.0092	(-0.87)
$Ancil. \times APH$	π_2	.0065	(2.74)
$Prof. \times APH$	π_3	.0043	(0.66)
$Admin. \times APH$	π_4	-.0012	(-0.49)
$Gen. \times APH$	π_5	-.0026	(-0.28)
$DS \times APH$	π_6	.0022	(0.31)
$CAP \times APH$	α_{KA}	-.0040	(-0.14)
Proprietary	d_p	-.1568	(-3.36)
Teaching	d_t	.0328	(0.92)

Table 3 Estimation Results
Cowing T.G., Holtmann A.G. (1983)

Having estimated equation (5.17) Cowing & Holtmann find that the hospitals are not in their long-run equilibrium: 130 of 138 hospitals are confronted with over-capitalization because the estimated coefficient for the variable capital is positive, whereas equation (5.15) requires the coefficient to be negative. In addition, the estimated coefficient for the number of admitting physicians is positive, meaning that reducing the number of physicians will result in lower costs. This can also be seen in equation (5.16), where a negative coefficient is required. Nevertheless, table 3 shows that the coefficient for the variable *number of admitting physicians* is statistically insignificant.

Cowing & Holtmann as well as Butler (see section 5.1.1) find that profit hospitals have lower costs compared to non-profit hospitals (see variable proprietary). They also confirm Butler's finding that the teaching status is not important if estimating hospital cost functions. The regression output from Cowing & Holtmann also show statistically significant parameter estimates for 3 of 5 diagnostic output categories and 2 significant coefficient estimates for the output categories in squared terms. On the other hand table 3 indicates that none of the parameter estimates for the cross products of the output categories are different from zero.

The authors also state, that all parameter estimates for the input prices are statistically highly significant just as most of the coefficients for the squared terms of the input prices. 8 of 15 coefficient estimates for the cross products of the input prices are different from zero. Contrary to the estimated coefficient from capital, the number

of admitting physicians has no significant influence on the costs. There are only a few significant coefficient estimates for the cross products of the output categories and the input prices. Cowing & Holtmann can not find significant parameter estimates for the cross products of the output categories or capital, and also no significant estimates for the cross products of the output categories or the number of admitting physicians.

Sinay & Campbell⁶⁸ also estimate a short-run translog variable cost function for 202 hospitals which are going to be subject to a merger one year later (merging hospitals), and for another 202 hospitals where this was not going to be the case (control hospitals). The authors state that the control hospitals are comparable with the merging hospitals in both size and ownership and are furthermore located in the same market area. Since most of the hospitals involved in mergers are located in or around the same local areas the last attribute is of particular importance, though the available data in the U.S. for the years 1987 – 1989 are taken from a nationwide sample. Just as Cowing & Holtmann, the authors use patient days as their unit of output, whereas Butler uses the number of cases treated.

Sinay & Campbell use the following short-run translog variable cost function for their analysis:

$$\begin{aligned} \ln C^V = & \alpha_0 + \sum_i \alpha_i \ln Y_i + \sum_i \sum_r \alpha_{ir} \ln Y_i \ln Y_r + \sum_j \beta_j \ln W_j \\ & + \sum_i \sum_j \beta_{ij} \ln W_i \ln W_j + \sum_i \sum_j \phi_{ij} \ln Y_i \ln W_j + \phi_K \ln BEDS \\ & + \frac{1}{2} \phi_{KK} (\ln BEDS)^2 + \sum_i \mu_i \ln Y_i \ln BEDS + \sum_j \delta_j \ln W_j \ln BEDS \\ & + \beta_{ser} SERV MIX + \beta_{pro} PROFIT + \beta_{sys} SYSTEM + d_1 DUMMY86 \\ & + d_2 DUMMY87 + d_3 DUMMY88 + e \end{aligned} \quad (5.19)$$

Y denotes the set of patient care output: Acute care days, intensive care days, sub acute care days, and outpatient visits. W reflects the input prices of labor and supplies, and $BEDS$ are the number of available beds which should represent the fixed capital. $SERV MIX$ is an index to account for the different services offered by the hospital and $SYSTEM$ is a dummy variable to identify if the given hospital is a member in a multi-hospital system. $PROFIT$ is a dummy variable for the proprietary status and $DUMMY86 - DUMMY88$ are dummy variables to control for differences in total variable costs over time.

Sinay & Campbell also introduce parameter restrictions to obtain a variable cost function being homogeneous of degree 1:

$$\sum \beta_j = 1, \quad \sum \beta_{ij} = 0, \quad \sum \phi_{ij} = 0, \quad \sum \delta_j = 0 \quad (5.20)$$

Their parameter estimates for merging and control hospitals are shown in table 4:

⁶⁸ Sinay U., Campbell C.: Scope and Scale Economies in Merging Hospitals Prior to Merger (1995)

MERGING HOSPITALS			CONTROL HOSPITALS		
VARIABLE	PARAMETER	ESTIMATE	t-STAT	ESTIMATE	t-STAT
Constant	α_0	.0006	.02	.0512	1.64*
Totacu	α_1	.3356	6.06**	.5660	12.26**
Totinte	α_2	.0961	5.05**	.1139	6.05**
Totsub	α_3	.0749	5.22**	.1076	6.83**
Totout	α_4	.1589	6.97**	.1268	6.38**
(Totacu) ²	α_{11}	.0294	2.78**	.0523	3.51**
(Totinte) ²	α_{22}	.0085	3.95**	.0184	4.31**
(Totsub) ²	α_{33}	.0061	4.32**	.0128	4.96**
(Totout) ²	α_{44}	.1948	1.10	.5633	4.37**
Totacu*totinte	α_{12}	-.0088	-.88	.0064	.61
Totacu*totsub	α_{13}	-.0544	-5.19**	-.0683	-7.01**
Totacu*totout	α_{14}	.0573	2.81**	.0081	1.08
Totinte*totsub	α_{23}	-.0023	-1.81*	.0010	.33
Totinte*totout	α_{24}	-.0014	-.25	-.0359	-3.98**
Totsub*totout	α_{34}	.0086	2.78**	.0062	1.40
Wage	β_1	.6213	141.5**	.6181	105.6**
Supplies	β_2	.3787	-	.3819	-
(Wage) ²	β_{11}	.1704	18.7**	.1592	13.8**
(Supplies) ²	β_{22}	-.0461	-5.15**	-.0387	-3.53**
Wage*supplies	β_{12}	-.1243	-	-.1205	-
Wage*totacu	ϕ_{11}	-.0090	-2.59**	.0007	.14
Wage*totinte	ϕ_{12}	.0020	1.75*	-.0070	-2.23**
Wage*totsub	ϕ_{13}	-.0007	-1.16	-.0056	-3.79**
Wage*totout	ϕ_{14}	.0086	3.36**	.0266	6.30**
Supplies*totacu	ϕ_{21}	-.0027	-.30	-.0090	-.58
Supplies*totinte	ϕ_{22}	-.0011	-.25	.0134	1.13
Supplies*totsub	ϕ_{23}	-.0200	-4.49**	-.0351	-5.36**
Supplies*totout	ϕ_{24}	.0229	-	.0159	-

MERGING HOSPITALS			CONTROL HOSPITALS		
VARIABLE	PARAMETER	ESTIMATE	t-STAT	ESTIMATE	t-STAT
Beds	ϕ_k	.3837	4.86**	.0398	.56
(Beds) ²	ϕ_{kk}	.0478	.74	-.1716	-4.90**
Totacu*beds	μ_1	-.0832	-1.43	.0159	.43
Totinte*beds	μ_2	.0100	.81	.0183	.91
Totsub*beds	μ_3	.0507	4.40**	.0485	3.56**
Totout*beds	μ_4	-.1110	-3.29**	-.0931	-3.98**
Beds*wage	δ_{k1}	.0034	.56	.0011	.12
Beds*supplies	δ_{k2}	-.0034	-	-.0011	-
Servmix	B_{scr}	-.1233	-1.03	.1344	1.16
Profit	B_{pro}	-.0270	-.93	-.0253	-.94
System	B_{sys}	.0111	.62	.0026	.16
Box-Cox P.	τ	.0901	5.57**	.1670	11.3**
Dummy86	d_1	-.0090	-.35	-.0511	-2.07**
Dummy87	d_2	.0372	1.40	-.0479	-2.04**
Dummy88	d_3	.0157	.58	-.0124	-.50
R ²		.983		.984	

* statistically significant at 10 percent level. ** statistically significant at 5 percent level.

Table 4 Parameter Estimates
Sinay U., Campbell C. (1995)

In their analysis the authors find that the parameter estimates for all 4 output categories are highly statistically significant for merging and control hospitals. Also the coefficient estimates for the squared terms of the output categories are nearly all relevant for both hospital types. Moreover, parameter estimates for the cross products of the diverse output rubrics are mostly statistically significant for merging hospitals. In addition, the input prices in linear and squared terms seem to be important when estimating a variable cost function. Also the estimates for the variables *wage* · *different output categories* are nearly all statistically significant for merging and control hospitals whereas the estimates for *supplies* · *output rubrics* seem to play no significant role in the estimation process.

The estimated positive coefficient for beds is highly significant for merging hospitals but not for the control hospitals whereas the coefficient for *beds*² is only relevant for control hospitals. The estimated coefficients for the variables *sub acute care days* · *BEDS* and *outpatient visits* · *BEDS* are significant at a 5 % level for both hospital types but all other variables including *BEDS* are not relevant for the analysis.

The coefficients for the variables *SERVMIX*, *PROFIT*, and *SYSTEM* are all statistically insignificant for both hospital types. Furthermore, there are only two significant coefficients for the year dummy variables of the control hospitals.

The estimate regression from Sinay & Campbell show an R^2 of more than 98 % for merging and control hospitals.

Sinay & Campbell also investigate whether or not economies of scale apply to the estimated variable cost function⁶⁹. For their analysis they classify merging and control hospitals into 3 sizes: mean size, large size (2 times mean size), and small size (0,5 times mean size).

The authors find that merging mean sized hospitals are confronted with diseconomies of scale whereas for the mean sized control hospitals economies of scale occur. As a result one can conclude that merging (mean sized) hospitals can become more efficient if they will proportionately reduce all outputs. Furthermore, it indicates that these hospitals do not benefit from the larger size after the merger.

For large hospitals, both types – merging and control hospitals – show diseconomies of scale. Small hospitals on the other hand show economies of scale with higher scale effects for control hospitals.

Sinay⁷⁰ also estimates a similar translog variable cost function just as the one shown under (5.19). Instead of year dummy variables, he includes 6 regional dummy variables, a variable for the year trend and a variable for the market size subject to the hospital's location. The major difference between this approach and the one given under (5.19) is the time frame of investigation: In addition to data 1 year before the merger, Sinay estimates the variable cost function for merging and control hospitals based on data generated 1 and 2 years after the merger. The estimation results for the merging and control hospitals are shown in table 5 and table 6:

As regards the coefficient estimates of the output categories for merging and control hospitals these estimation results match those described above. In contrast, the coefficients of the output rubrics in squared terms estimated by Sinay and shown in table 5 (control hospitals) differ in significance from those estimated by Sinay & Campbell. The parameter estimates for the cross products of the output categories are mostly not statistically significant for merging hospitals but significant for control hospitals 1 year before the merger and 1 year after a merger. To include the inputs in linear and squared terms into the regression analysis seems to be as relevant in this case as it is for the estimation output of Sinay & Campbell. This applies also to the coefficient of beds, which is again statistically significant, this time for both hospital types. However, it is remarkable that the coefficient turns out to be negative only for merging hospitals after the merger. It can be seen in table 6 that all variables including beds are now more important for control hospitals than for the hospitals studied by Sinay & Campbell (see table 4). The dummy variables which account for

⁶⁹ See chapter 3 for the calculation procedure

⁷⁰ Sinay U.T.: Pre- and Post-Merger Investigation of Hospital Mergers (1998)

SERVMIX, *PROFIT*, and *SYSTEM* show no statistical significance for the merging hospitals. On the other hand the coefficient for *SERVMIX* is now statistically significant for the control hospitals.

VARIABLE	PARAMETER	Pre-Merger ESTIMATE	One Year After ESTIMATE	Two Years After ESTIMATE
Constant	α_0	-0.0114	-0.0067	-0.2496 ^a
Totacu	α_1	0.3237 ^a	0.7586 ^a	0.7415 ^a
Totinte	α_2	0.1244 ^a	0.1580 ^a	0.1464 ^a
Totsub	α_3	0.0634 ^a	0.1721 ^a	0.1606 ^a
Totout	α_4	0.1432 ^a	0.0841 ^a	0.0624 ^b
(Totacu) ²	α_{11}	0.0533 ^a	-0.3483 ^b	0.1657
(Totinte) ²	α_{22}	0.0122 ^a	0.0290	-0.0049
(Totsub) ²	α_{33}	0.0057 ^a	0.0224 ^a	0.0468 ^a
(Totout) ²	α_{44}	0.4516	-0.4996	-0.4390 ^a
Totacu×totinte	α_{12}	-0.0244	-0.0217	-0.0660
Totacu×totsub	α_{13}	-0.0550 ^a	-0.1849 ^a	-0.2557 ^a
Totacu×totout	α_{14}	0.0375	0.1552	0.3373 ^a
Totinte×totsub	α_{23}	-0.0026	-0.0203	-0.0329
Totinte×totout	α_{24}	-0.0123	-0.0201	0.0018
Totsub×totout	α_{34}	0.0078 ^b	-0.0206	0.0130
Wage	β_1	0.6438 ^a	0.6330 ^a	0.6381 ^a
Supplies	β_2	0.3562 ^a	0.3670 ^a	0.3619 ^a
(Wage) ²	β_{11}	0.1762 ^a	0.1944 ^a	0.1763 ^a
(Supplies) ²	β_{22}	-0.0500 ^a	-0.0402 ^a	-0.0033
Wage×supplies	β_{12}	-0.1262 ^a	-0.1542 ^a	-0.1796
Wage×totacu	ϕ_{11}	-0.0149 ^a	0.0286 ^b	0.0116
Wage×totinte	ϕ_{12}	0.0038	0.0025	0.0159 ^a
Wage×totsub	ϕ_{13}	-0.0001	-0.0063	-0.0228 ^a
Wage×totout	ϕ_{14}	0.0241 ^a	0.0185 ^b	0.0091
Supplies×totacu	ϕ_{21}	-0.0466	0.2079 ^b	-0.2359 ^a
Supplies×totinte	ϕ_{22}	0.0182	-0.0758	0.1276 ^a
Supplies×totsub	ϕ_{23}	-0.0297 ^a	-0.0907 ^a	0.0088
Supplies×totout	ϕ_{24}	0.0452	-0.0847	0.0857
Beds	φ_k	0.3974 ^a	-0.1773 ^b	-0.2529 ^a
(Beds) ²	φ_{kk}	0.0002	-0.5908 ^a	-0.2028
Totacu×beds	μ_1	-0.0610	0.5422	-0.3886 ^b
Totinte×beds	μ_2	0.0174	0.1449	0.1959 ^a
Totsub×beds	μ_3	0.0579 ^a	0.2399 ^a	0.2569 ^a
Totout×beds	μ_4	-0.0840 ^b	-0.1727	-0.3570 ^a
Beds×wage	δ_{k1}	-0.0011	-0.0457 ^a	-0.0161
Beds×supplies	δ_{k2}	0.0011	0.0457 ^a	0.0161
Servmix	B_{ser}	0.0470	-0.0227	0.0645
Profit	B_{pro}	-0.0146	0.0170	-0.0801
System	B_{sys}	0.0159	-0.0131	0.0172
Box-Cox P.	τ	0.1146 ^a	0.3161 ^a	0.4417 ^a
Year trend	t	-0.0080	0.0163 ^b	0.0062
Metro size	m	0.0130	-0.0131	0.0387 ^a
Region 2	r_2	-0.0877 ^a	0.0425	0.1006 ^a
Region 3	r_3	-0.1840 ^a	0.1563 ^a	0.0186
Region 4	r_4	-0.1502 ^a	-0.0747	-0.0087
Region 5	r_5	-0.0178	0.0275	0.0685
Region 6	r_6	-0.1488 ^a	0.0272	0.0289
R ²		0.985	0.989	0.986

a. Statistically significant at the 5 percent level.
b. Statistically significant at the 10 percent level.

Table 5 Regression Results for Merging Hospitals
Sinay U.T. (1998)

VARIABLE	PARAMETER	Pre-Merger ESTIMATE	One Year After ESTIMATE	Two Years After ESTIMATE
Constant	α_0	-0.1552 ^a	-0.3563 ^a	-0.1744 ^a
Totacu	α_1	0.4373 ^a	0.4602 ^a	0.3800 ^a
Totinte	α_2	0.0923 ^a	0.0793 ^a	0.1199 ^a
Totsub	α_3	0.0568 ^a	0.1035 ^a	0.1116 ^a
Totout	α_4	0.1361 ^a	0.1205 ^a	0.0763 ^a
(Totacu) ²	α_{11}	0.0309	0.2211 ^a	0.0248
(Totinte) ²	α_{22}	0.0258 ^a	-0.0057	0.0157 ^b
(Totsub) ²	α_{33}	0.0052	0.0180 ^a	0.0108 ^a
(Totout) ²	α_{44}	0.2196	0.4619 ^b	-0.3530
Totacu×totinte	α_{12}	-0.0566 ^a	-0.1707 ^a	-0.0379
Totacu×totsub	α_{13}	-0.0821 ^a	-0.0848 ^a	-0.1125 ^a
Totacu×totout	α_{14}	0.0560 ^a	-0.1186 ^a	0.0712 ^a
Totinte×totsub	α_{23}	-0.0151 ^a	-0.0594 ^a	0.0081
Totinte×totout	α_{24}	-0.0211	0.0525 ^b	-0.0335
Totsub×totout	α_{34}	0.0129 ^b	0.0175	0.0026
Wage	β_1	0.6441 ^a	0.6389 ^a	0.6382 ^a
Supplies	β_2	0.3559 ^a	0.3611 ^a	0.3618 ^a
(Wage) ²	β_{11}	0.1327 ^a	0.1414 ^a	0.1541 ^a
(Supplies) ²	β_{22}	-0.0326 ^a	-0.0322 ^a	-0.0662 ^a
Wage×supplies	β_{12}	-0.1001 ^a	-0.1092 ^a	-0.0879 ^a
Wage×totacu	ϕ_{11}	0.0112	-0.0131 ^b	-0.0028
Wage×totinte	ϕ_{12}	0.0041	0.0128 ^a	0.0011
Wage×totsub	ϕ_{13}	-0.0019	-0.0078 ^a	-0.0035 ^a
Wage×totout	ϕ_{14}	0.0248 ^a	0.0227 ^a	0.0207 ^a
Supplies×totacu	ϕ_{21}	-0.0203	-0.1333 ^a	-0.1476 ^a
Supplies×totinte	ϕ_{22}	-0.0150	0.0498	0.0568 ^b
Supplies×totsub	ϕ_{23}	-0.0171 ^a	0.0202 ^a	-0.0303 ^a
Supplies×totout	ϕ_{24}	0.0142	0.0487	0.1056
Beds	φ_k	0.1954 ^a	0.1451	0.2272 ^a
(Beds) ²	φ_{kk}	-0.2836 ^a	-0.5153 ^a	-0.4103 ^a
Totacu×beds	μ_1	0.1782 ^a	0.1129	0.2654 ^a
Totinte×beds	μ_2	0.0687	0.2700 ^a	0.0232
Totsub×beds	μ_3	0.0947 ^a	0.1458 ^a	0.1282 ^a
Totout×beds	μ_4	-0.2099 ^a	-0.1304 ^a	-0.0605
Beds×wage	δ_{k1}	-0.0360 ^a	-0.0137	-0.0166 ^b
Beds×supplies	δ_{k2}	0.0360 ^a	0.0137	0.0166 ^b
Servmix	B_{ser}	0.1208 ^a	0.1591 ^a	0.1280 ^a
Profit	B_{pro}	0.0542	0.0360	0.0521
System	B_{sys}	-0.0079	-0.0245	-0.0126
Box-Cox P.	τ	0.2093 ^a	0.2388 ^a	0.1792 ^a
Year trend	t	0.0015	0.0378 ^a	0.0143
Metro size	m	0.0307 ^a	0.0115	-0.0117
Region 2	r_2	-0.0400	0.0762 ^a	-0.0197
Region 3	r_3	-0.0488	-0.0056	-0.0644
Region 4	r_4	0.0155	-0.0133	0.0491 ^a
Region 5	r_5	0.0109	0.0629	0.0563
Region 6	r_6	0.0370	0.0672 ^b	0.0250
R ²		0.989	0.986	0.987

a. Statistically significant at the 5 percent level.

b. Statistically significant at the 10 percent level.

Table 6 Regression Results for Control Hospitals
Sinay U.T. (1998)

Wang, Zhao & Mahmood⁷¹ estimate a translog variable cost function for large and small hospitals (i.e. district hospitals). For their analysis they use data from 114 public hospitals in New South Wales (Australia) from the years 1997 – 1998.

Their estimate translog variable cost function includes I variable inputs, T fixed factors and N outputs and can be described as follows

$$\begin{aligned} \ln C^{V*} = & \alpha_0 + \sum_{i=1}^I \alpha_i \ln w_i^* + \sum_{n=1}^N \beta_n \ln y_n^* + \sum_{t=1}^T \gamma_t \ln k_t^* \\ & + \frac{1}{2} \sum_{i=1}^I \sum_{j=1}^I \alpha_{ij} \ln w_i^* \ln w_j^* + \frac{1}{2} \sum_{n=1}^N \sum_{m=1}^N \beta_{nm} \ln y_n^* \ln y_m^* \\ & + \frac{1}{2} \sum_{t=1}^T \sum_{s=1}^T \gamma_{ts} \ln k_t^* \ln k_s^* + \sum_{i=1}^I \sum_{n=1}^N \rho_{in} \ln w_i^* \ln y_n^* \\ & + \sum_{i=1}^I \sum_{t=1}^T \delta_{it} \ln w_i^* \ln k_t^* + \sum_{n=1}^N \sum_{t=1}^T \theta_{nt} \ln y_n^* \ln k_t^* \\ & + \vartheta x + v + u \end{aligned} \quad (5.21)$$

where w_i^* represents the input prices (i.e. the average salary of medical labor services and the average salary of non-medical labor inputs), y_n^* denotes the output levels, and k_t^* describes the fixed factors (i.e. the number of average available beds). x is a vector which consists of hospital complexity indicators, v is the included error term for the output variables, and u includes other random factors.

The authors also restrict the following parameters to get a well defined cost function, which has to fulfill the properties mentioned at the beginning of chapter 3.

For a continuous cost function in the factor prices and output levels the following restrictions are given:

$$\begin{aligned} \alpha_{ij} &= \alpha_{ji} \quad \forall i, j \\ \beta_{nm} &= \beta_{mn} \quad \forall n, m \end{aligned} \quad (5.22)$$

The request for a homogeneous cost function of degree 1 demands the following restrictions:

$$\begin{aligned} \sum_{i=1}^I \alpha_i &= 1, \quad \sum_{j=1}^I \alpha_{ij} = 0, \quad i = 1, \dots, I \\ \sum_{i=1}^I \rho_{in} &= 0, \quad n = 1, \dots, N \\ \sum_{i=1}^I \delta_{it} &= 0, \quad t = 1, \dots, T \end{aligned} \quad (5.23)$$

In order to obtain a cost function which is nondecreasing in factor prices and output levels, the following properties are given:

$$\begin{aligned} \alpha_i &\geq 0, \quad i = 1, \dots, I \\ \beta_n &\geq 0, \quad n = 1, \dots, N \end{aligned} \quad (5.24)$$

Wang, Zhao & Mahmood formulate an index for the number of inpatient services and an index for the number of occasions of services which they use as an approximation for outpatient care. The authors apply these two indices to measure the hospital outputs. Therefore they define the case mix inpatient service index as follows:

⁷¹ Wang J., Zhao Z., Mahmood A.: Relative Efficiency, Scale Effect, and Scope Effect of Public Hospitals: Evidence from Australia (2006)

$$M_h = \sum_d \frac{x_{dh}}{x_h} \cdot \bar{s}_d \quad (5.25)$$

Wang, Zhao & Mahmood denote X as the number of separations, h is a dummy indicator of large or small hospitals, and d is an index for the diagnostic category. \bar{s}_d acts as a weight reflecting the average length of stay for separations with conditions given by the indices of the diagnostic category.

Looking at the regression output, one has to notice that most of the estimated coefficients are statistically insignificant for large and small hospitals. The only estimated coefficients which are statistically significant for small hospitals are for the variables *salary of non – medical labour inputs*, *total occasions of services*, and *total occasions of services · average available beds*. The regression output for large hospitals shows a similar paradigm while there are a few more significant parameter estimates⁷² (e.g.: average available beds, total occasions of services in squared terms, salary of non-medical labor inputs times average available beds). Due to the fact that the estimated coefficient for the variable *average available beds* is positive and significant, the authors conclude, that in large hospitals over-capitalization occurs.

Considering the numerous statistically insignificant coefficient estimates, it can be expected that the resulting R^2 of 96 % for large hospitals and 95% for small hospitals are only due to the large number of explanatory variables included.

The authors conclude that the translog variable cost function for small hospitals is U-shaped and shows economies of scale until the minimum point of 43 beds is reached. The translog variable cost function for large hospitals is also U-shaped with the minimum at 175 beds. For both hospital types economies of scope are present.

5.2 Conclusion

As can clearly be seen in the current chapter, numerous authors come to different conclusions which variables should be included into a hospital cost function (especially when focusing on cross products). Furthermore, the cost function chosen for any estimation procedure will not be subject to common consent. There are authors who prefer to estimate an ad-hoc specification whereas others concentrate on flexible functional forms. However, looking at the regression outputs described above it can be seen, that there is no disagreement about the question whether different output categories and input prices shall be included into a hospital cost function. By including fixed factors into the cost function (e.g.: the number of available beds) the regression results can be improved.

Having investigated the relevant factors to be included into a hospital cost function (i.e. output categories in linear and squared terms including the cross products; the input prices in linear and squared terms and the cross products of the input prices with the output categories; the number of available beds) I can now continue in chapter 6 to conduct the merger analysis of the hospital market.

⁷² 7 parameter estimates of 27 are statistically significant.

6 Merger Analysis of the Hospital Sector

In the previous chapters all aspects providing the base of a horizontal merger analysis of the hospital sector have been discussed. While horizontal mergers between single product firms were introduced in chapter 2, chapter 3 gave a brief recapitulation of cost functions of multiproduct firms such as hospitals. In chapters 4 and 5 the units of outputs and their measurement were defined, and the general layout of a hospital cost function was analyzed.

In this last chapter a merger analysis of the hospital sector will be given. The main focus of this analysis is on the changes in costs and profits which might occur as a result of a merger. The general approach is similar to the one in chapter 2, though in the current chapter the multiproduct case instead of the single product case will be analyzed. For the purpose of calculation a hospital cost function will be assumed.

As noted in chapter 2, I suppose that Cournot competition is an accurate method for describing the hospital market particularly in Europe, where a broad public health insurance system is available. Therefore hospitals and hospital wards are assumed to compete in quantities and not in prices.

In the following analysis I distinguish between two cases: A merger between hospital units and a merger between hospitals. For the first case – a merger between hospital wards – it is assumed that all three hospital units in the market produce exactly the same outputs (i.e. they are producing perfect substitutes). Furthermore it is assumed that hospital wards 2 and 3 are going to merge and that no capacity constraints exist. Since all three hospital units produce homogeneous outputs, only two of the three hospital wards will remain in the market after a merger has taken place.

A different situation occurs for two merging hospitals: In this case we may assume that all three hospitals in the market produce differentiated outputs (subject to their size, location, operator, level of specialization, etc). This means that each hospital in the market produces outputs with none of them being exactly equal to the others. Therefore, if a merger between hospitals 2 and 3 takes place a new hospital M , which will going to produce the different outputs of both hospitals, will arise.

The $(m \times n)$ output matrix of the hospital sector is given by:

$$Q = \begin{bmatrix} q_{11} & \dots & q_{1j} \dots & q_{1n} \\ \vdots & & \ddots & \vdots \\ q_{i1} & & & q_{in} \\ \vdots & & & \vdots \\ q_{m1} & \dots & q_{mj} \dots & q_{mn} \end{bmatrix} \quad (6.1)$$

$i = 1, \dots, m$, denotes the different hospitals or hospital wards respectively

$j = 1, \dots, n$, represents the different outputs

For simplicity I assume that only 3 hospitals or hospital units are present in the hospital market, so that there is still some competition after a merger. Furthermore, each hospital (unit) is producing 2 outputs. The output matrix of the hospital market therefore becomes a (3×2) matrix:

$$Q = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \\ q_{31} & q_{32} \end{bmatrix} \quad (6.2)$$

i.e.: $i = 1, 2, 3$, are the different hospitals or hospital units respectively
 $j = 1, 2$, describes the different outputs produced

In chapter 5 the statistical significance of various variables included in hospital cost functions is illustrated. As stated in the conclusion it is important to include the different output categories as well as the different input prices if working with a hospital cost function (both in linear and quadratic terms, and also the cross products). The cross products of the input prices with the different output categories shall be incorporated as well. Furthermore, the regression results can be improved by including fixed factors (e.g. the number of available beds) into the hospital cost function.

As described in chapter 5, Sinay⁷³ estimated hospital cost functions for merging and control hospitals for the periods 1 year before the merger as well as 1 and 2 years after the merger. Table 5 (regression results for merging hospitals) and table 6 (regression results for control hospitals) illustrate the fact that the estimated coefficients will change over these periods. Thus, we assume that cost functions which include different coefficients before and after the merger for merging and control hospitals respectively hospital wards exist. Moreover, to capture the coefficients' relationship before and after a merger as well as between the estimated coefficients, the mean values of the statistically significant coefficients (1 year before and 2 years after the merger) estimated by Sinay are used for the calculation⁷⁴.

6.1 Merger between Hospital Wards

Examining in particular wards in the hospital market, it can be assumed that they all produce exactly the same outputs. This means that each hospital unit provides treatments for a narrow area of diseases and hence specialization in the different hospital wards occurs.

Suppose that the inverse demand function has the following form:

$$P_j = a - \sum_{i=1}^3 q_{ij} \quad (6.3)$$

The linear invese demand function suggests that the price of output j depends on the quantity of output j produced in every hospital unit. Since Cournot competition is assumed in the hospital market, the function reveals the fact that in this oligopoly game quantities of different hospital wards are strategic substitutes. It follows, that if any given hospital unit in the market produces more of output j , the price of output j will fall. $a > 0$ is a constant which indicates that even if there is no treatment of output j , the price of output j is positive.

⁷³ Sinay U.T.: Pre- and Post-Merger Investigation of Hospital Mergers (1998)

⁷⁴ To fulfill the parameter restrictions $\sum \beta_j = 1$, $\sum \beta_{ij} = 0$, $\sum \phi_{ij} = 0$, $\sum \delta_j = 0$ (see chapter 5), also statistically insignificant coefficients are used to calculate the mean values. This is necessary to obtain a cost function which is homogeneous of degree 1.

The assumed cost functions for the hospital units are given as follows:

- **Cost Functions for Control Wards**

- Before the Merger:

$$\begin{aligned}
 C_{ij} = & 0,1806 q_{i1} + 0,1806 q_{i2} + 0,0065 q_{i1}^2 + 0,0065 q_{i2}^2 - 0,0142 q_{i1} q_{i2} \\
 & + 0,6441 w_{i1} + 0,3559 w_{i2} + 0,1327 w_{i1}^2 - 0,0326 w_{i2}^2 - 0,1001 w_{i1} w_{i2} \\
 & + 0,0096 q_{i1} w_{i1} + 0,0096 q_{i2} w_{i1} - 0,0096 q_{i1} w_{i2} - 0,0096 q_{i2} w_{i2} \\
 & + 0,1954 \text{ beds}
 \end{aligned} \tag{6.4}$$

- After the Merger:

$$\begin{aligned}
 C_{ij} = & 0,1720 q_{i1} + 0,1720 q_{i2} + 0,0066 q_{i1}^2 + 0,0066 q_{i2}^2 - 0,0069 q_{i1} q_{i2} \\
 & + 0,6382 w_{i1} + 0,3618 w_{i2} + 0,1541 w_{i1}^2 - 0,0662 w_{i2}^2 - 0,0879 w_{i1} w_{i2} \\
 & + 0,0039 q_{i1} w_{i1} + 0,0039 q_{i2} w_{i1} - 0,0039 q_{i1} w_{i2} - 0,0039 q_{i2} w_{i2} \\
 & + 0,2272 \text{ beds}
 \end{aligned} \tag{6.5}$$

- **Cost Functions for Merging Wards**

- Before the Merger:

$$\begin{aligned}
 C_{ij} = & 0,1637 q_{i1} + 0,1637 q_{i2} + 0,0178 q_{i1}^2 + 0,0178 q_{i2}^2 - 0,0079 q_{i1} q_{i2} \\
 & + 0,6438 w_{i1} + 0,3562 w_{i2} + 0,1762 w_{i1}^2 - 0,0500 w_{i2}^2 - 0,1262 w_{i1} w_{i2} \\
 & + 0,0032 q_{i1} w_{i1} + 0,0032 q_{i2} w_{i1} - 0,0032 q_{i1} w_{i2} - 0,0032 q_{i2} w_{i2} \\
 & + 0,3974 \text{ beds}
 \end{aligned} \tag{6.6}$$

- After the Merger:

$$\begin{aligned}
 C_{ij} = & 0,2777 q_{i1} + 0,2777 q_{i2} - 0,0981 q_{i1}^2 - 0,0981 q_{i2}^2 + 0,0136 q_{i1} q_{i2} \\
 & + 0,6381 w_{i1} + 0,3619 w_{i2} + 0,17623 w_{i1}^2 - 0,0033 w_{i2}^2 - 0,1796 w_{i1} w_{i2} \\
 & + 0,0035 q_{i1} w_{i1} + 0,0035 q_{i2} w_{i1} - 0,0035 q_{i1} w_{i2} - 0,0035 q_{i2} w_{i2} \\
 & - 0,2529 \text{ beds}
 \end{aligned} \tag{6.7}$$

The variable *beds* represents the number of staffed beds in the hospital unit and shall reflect the fixed capital. The variable w_{i1} describes the average salary, and w_{i2} denotes the average price of supplies for hospital unit i . q_{i1} and q_{i2} are the two possible outputs each hospital ward is going to produce. The major difference to the single product case is the interaction term $q_{i1} \cdot q_{i2}$ where a negative coefficient indicates that synergies due to the joint production are present.

Furthermore, it is assumed that a merger between hospital units 2 and 3 takes place (i.e. hospital unit 1 is a control ward; hospital units 2 and 3 are merging wards).

6.1.1 The Pre-Merger Situation

As stated above it is assumed that hospitals and hospital units compete in quantities. Therefore hospital unit $i = 1, 2, 3$ has to maximize its profit function to obtain the equilibrium quantities it is going to produce:

$$\max. \pi_i = (a - \sum_{i=1}^3 q_{i1}) q_{i1} + (a - \sum_{i=1}^3 q_{i2}) q_{i2} - C_{ij} \quad (6.8)$$

The first order conditions are as follows (note that hospital unit 1 is a control ward, and hospital units 2 and 3 are merging wards):

- **Ward 1:**

$$\begin{aligned} \frac{\partial \pi_1}{\partial q_{11}} &= a - 2q_{11} - q_{21} - q_{31} - 0,1806 - 0,0013 q_{11} + 0,0142 q_{12} \\ &\quad - 0,0096 w_{11} + 0,0096 w_{12} = 0 \end{aligned} \quad (6.9)$$

$$\begin{aligned} \frac{\partial \pi_1}{\partial q_{12}} &= a - 2q_{12} - q_{22} - q_{32} - 0,1806 - 0,0013 q_{12} + 0,0142 q_{11} \\ &\quad - 0,0096 w_{11} + 0,0096 w_{12} = 0 \end{aligned} \quad (6.10)$$

- **Ward 2:**

$$\begin{aligned} \frac{\partial \pi_2}{\partial q_{21}} &= a - 2q_{21} - q_{11} - q_{31} - 0,1637 - 0,0036 q_{21} + 0,0079 q_{22} \\ &\quad - 0,0032 w_{21} + 0,0032 w_{22} = 0 \end{aligned} \quad (6.11)$$

$$\begin{aligned} \frac{\partial \pi_2}{\partial q_{22}} &= a - 2q_{22} - q_{12} - q_{32} - 0,1637 - 0,0036 q_{22} + 0,0079 q_{21} \\ &\quad - 0,0032 w_{21} + 0,0032 w_{22} = 0 \end{aligned} \quad (6.12)$$

- **Ward 3:**

$$\begin{aligned} \frac{\partial \pi_3}{\partial q_{31}} &= a - 2q_{31} - q_{11} - q_{21} - 0,1637 - 0,0036 q_{31} + 0,0079 q_{32} \\ &\quad - 0,0032 w_{31} + 0,0032 w_{32} = 0 \end{aligned} \quad (6.13)$$

$$\begin{aligned} \frac{\partial \pi_3}{\partial q_{32}} = & a - 2q_{32} - q_{12} - q_{22} - 0,1637 - 0,0036 q_{32} + 0,0079 q_{31} \\ & - 0,0032 w_{31} + 0,0032 w_{32} = 0 \end{aligned} \quad (6.14)$$

To solve these first order conditions, symmetry on quantities within each hospital unit is assumed. This means, that each hospital unit is producing the same amount of its two products, i.e.:

$$q_{i1} = q_{i2} \quad (6.15)$$

After rewriting and solving the first order conditions, the following equilibrium quantities are obtained:

$$\begin{aligned} q_{11}^* = q_{12}^* = & 0,2550 a + 0,0009 w_{21} - 0,0009 w_{22} + 0,0009 w_{31} - 0,0009 w_{32} \\ & - 0,0073 w_{11} + 0,0073 w_{12} - 0,0523 \end{aligned} \quad (6.16)$$

$$\begin{aligned} q_{21}^* = q_{22}^* = & 0,2460 a - 0,0024 w_{21} + 0,0024 w_{22} + 0,0008 w_{31} - 0,0008 w_{32} \\ & + 0,0024 w_{11} - 0,0024 w_{12} - 0,0368 \end{aligned} \quad (6.17)$$

$$\begin{aligned} q_{31}^* = q_{32}^* = & 0,2460 a + 0,0008 w_{21} - 0,0008 w_{22} - 0,0024 w_{31} + 0,0024 w_{32} \\ & + 0,0024 w_{11} - 0,0024 w_{12} - 0,0368 \end{aligned} \quad (6.18)$$

The equilibrium price of output j in the pre-merger period is given by:

$$P_j^* = a - \sum_{i=1}^3 q_{ij}^* = a - q_{1j}^* - q_{2j}^* - q_{3j}^* \quad (6.19)$$

$$\begin{aligned} P_j^* = & 0,2530 a + 0,0007 w_{21} - 0,0007 w_{22} + 0,0007 w_{31} - 0,0007 w_{32} \\ & + 0,0025 w_{11} - 0,0025 w_{12} + 0,1259 \end{aligned} \quad (6.20)$$

The total output of the three hospital wards is as follows:

$$Q^* = 2 \cdot q_{1j}^* + 2 \cdot q_{2j}^* + 2 \cdot q_{3j}^* \quad (6.21)$$

$$\begin{aligned} Q^* = & 1,4940 a - 0,0015 w_{21} + 0,0015 w_{22} - 0,0015 w_{31} + 0,0015 w_{32} \\ & - 0,0050 w_{11} + 0,0050 w_{12} - 0,2518 \end{aligned} \quad (6.22)$$

For a better comparison between the pre- and post-merger situation as well as for the sake of a simpler calculation of the cost and profit functions, concrete values for

w_{i1} , w_{i2} , and $beds$ will be used. Therefore, I suppose that the number of staffed beds in a hospital unit equals 30. Furthermore, Sinay⁷⁵ notes, that the mean value of the supply price per patient day is \$134 for merging hospitals and \$129 for control hospitals. He also suggests that the mean value of the salary per employee per year is \$22.664 for merging hospitals and \$21.845 for control hospitals. After some calculations⁷⁶ to obtain variables with the same base (namely patient days), the following values are used for the further analysis⁷⁷:

$$\begin{aligned} w_{11} &= \$ 233,9; w_{12} = \$ 129 \\ w_{21} &= \$ 243,6; w_{22} = \$ 134 \\ w_{31} &= \$ 243,6; w_{32} = \$ 134 \end{aligned} \quad (6.23)$$

Thus, all the equilibrium values given above can be rewritten as follows:

$$\begin{aligned} q_{11}^* &= q_{12}^* = 0,2550 a - 0,6220 \\ q_{21}^* &= q_{22}^* = 0,2460 a + 0,0363 \\ q_{31}^* &= q_{32}^* = 0,2460 a + 0,0363 \end{aligned} \quad (6.24)$$

$$P_j^* = 0,2530 a + 0,5490 \quad (6.25)$$

$$Q^* = 1,4940 a - 1,0981 \quad (6.26)$$

Using concrete values for the average salary, the average supply price and the number of staffed beds, the cost and profit functions of the 3 hospital units can also easily be calculated:

$$\begin{aligned} C_{1j}^* &= -0,0001 a^2 + 0,6033 a + 3.898,04 \\ C_{2j}^* &= 0,0017 a^2 + 0,2540 a + 5.655,13 \\ C_{3j}^* &= 0,0017 a^2 + 0,2540 a + 5.655,13 \end{aligned} \quad (6.27)$$

$$\begin{aligned} \pi_1 &= 0,1291 a^2 - 0,6381 a - 3.898,04 \\ \pi_2 &= 0,1228 a^2 + 0,0344 a - 5.655,13 \\ \pi_3 &= 0,1228 a^2 + 0,0344 a - 5.655,13 \end{aligned} \quad (6.28)$$

⁷⁵ Sinay U.T.: Pre- and Post-Merger Investigation of Hospital Mergers (1998)

⁷⁶ Sinay indicates that the supply price is per patient day, whereas the salary is given per employee per year. It is known that (total supply prices / total patient days) = 134, so the total patient days can be determined since the total supply prices are given in his paper. Having obtained nearly 7.000 patient days, and given the total wage costs of \$ 16.825.788 for merging hospitals and \$ 16.037.532 for control hospitals, the average salary per patient day is \$ 243,6 for merging hospitals and \$ 233,9 for control hospitals.

⁷⁷ It can be expected that the average salary and the average supply price don't differ significantly before and after the merger. Thus the same values are used in both situations.

Having investigated the pre-merger situation, the next subsection describes the post-merger scenario followed by a comparison of these two situations.

6.1.2 The Post-Merger Situation

Suppose a merger between hospital units 2 and 3 occurs and no capacity constraints exist. Since all hospital wards produce exactly the same outputs, only hospital units 1 and 2 persist in the market, whereas hospital unit 3 does not exist any more.

The output matrix of the hospital market is then given by:

$$Q = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \quad (6.29)$$

In the further analysis $i = 1, 2$, denotes the number of hospital wards, and $j = 1, 2$, represents the two different outputs.

As in the pre-merger situation, each hospital unit is trying to maximize its profits by choosing the optimal quantities. Therefore, the following profit function has to be maximized:

$$\max_i \pi_i = (a - \sum_{i=1}^2 q_{i1}) q_{i1} + (a - \sum_{i=1}^2 q_{i2}) q_{i2} - C_{ij} \quad (6.30)$$

To obtain the equilibrium values of the quantities that shall be produced, one again has to calculate the first order conditions for hospital units 1 and 2 (note that hospital unit 1 is a control ward, and hospital unit 2 is a merging ward):

- **Ward 1:**

$$\begin{aligned} \frac{\partial \pi_1}{\partial q_{11}} &= a - 2q_{11} - q_{21} - 0,1720 - 0,0132 q_{11} + 0,0069 q_{12} \\ &\quad - 0,0039 w_{11} + 0,0039 w_{12} = 0 \end{aligned} \quad (6.31)$$

$$\begin{aligned} \frac{\partial \pi_1}{\partial q_{12}} &= a - 2q_{12} - q_{22} - 0,1720 - 0,0013 q_{12} + 0,0069 q_{11} \\ &\quad - 0,0039 w_{11} + 0,0039 w_{12} = 0 \end{aligned} \quad (6.32)$$

- **Ward 2:**

$$\begin{aligned} \frac{\partial \pi_2}{\partial q_{21}} &= a - 2q_{21} - q_{11} - 0,2777 + 0,1962 q_{21} - 0,0136 q_{22} \\ &\quad - 0,0035 w_{21} + 0,0035 w_{22} = 0 \end{aligned} \quad (6.33)$$

$$\begin{aligned} \frac{\partial \pi_2}{\partial q_{22}} = a - 2q_{22} - q_{12} - 0,2777 + 0,1962 q_{22} - 0,0136 q_{21} \\ - 0,0035 w_{21} + 0,0035 w_{22} = 0 \end{aligned} \quad (6.34)$$

As in the pre-merger situation, it is assumed that each hospital unit produces the same amount of output 1 and 2, i.e.:

$$q_{i1} = q_{i2} \quad (6.35)$$

Due to the symmetry condition under (6.35) the first order conditions can be solved, and the equilibrium quantities will be obtained:

$$\begin{aligned} q_{11}^* = q_{12}^* = 0,3094 a + 0,0013 w_{21} - 0,0013 w_{22} \\ - 0,0027 w_{11} + 0,0027 w_{12} - 0,0133 \end{aligned} \quad (6.36)$$

$$\begin{aligned} q_{21}^* = q_{22}^* = 0,380 a - 0,0026 w_{21} + 0,0026 w_{22} \\ + 0,0015 w_{11} - 0,0015 w_{12} - 0,1455 \end{aligned} \quad (6.37)$$

The equilibrium price of output j in the post-merger situation is as follows:

$$P_j^* = a - \sum_{i=1}^2 q_{ij}^* = a - q_{1j}^* - q_{2j}^* \quad (6.38)$$

$$\begin{aligned} P_j^* = 0,3106 a + 0,0013 w_{21} - 0,0013 w_{22} \\ + 0,0012 w_{11} - 0,0012 w_{12} + 0,1588 \end{aligned} \quad (6.39)$$

The total industry output in the equilibrium is given by:

$$Q^* = 2 \cdot q_{1j}^* + 2 \cdot q_{2j}^* \quad (6.40)$$

$$\begin{aligned} Q^* = 1,3788 a - 0,0026 w_{21} + 0,0026 w_{22} \\ - 0,0024 w_{11} + 0,0024 w_{12} - 0,3176 \end{aligned} \quad (6.41)$$

Inserting the values for the average salary and the average supply price given under (6.23) the equilibrium quantities and the optimal price of output j can be rewritten:

$$\begin{aligned} q_{11}^* = q_{12}^* = 0,3094 a - 0,1562 \\ q_{21}^* = q_{22}^* = 0,3800 a - 0,2758 \end{aligned} \quad (6.42)$$

$$P_j^* = 0,3106 a + 0,4320 \quad (6.43)$$

$$Q^* = 1,3788 a - 0,2391 \quad (6.44)$$

Also the cost and profit functions of hospital wards 1 and 2 can be calculated:

$$\begin{aligned} C_{1j}^* &= 0,0006 a^2 + 0,3574 a + 4.879,43 \\ C_{2j}^* &= -0,0264 a^2 + 0,5368 a + 4.675,01 \end{aligned} \quad (6.45)$$

$$\begin{aligned} \pi_1 &= 0,1916 a^2 - 0,1871 a - 4.879,43 \\ \pi_2 &= 0,2625 a^2 - 0,3798 a - 4.675,01 \end{aligned} \quad (6.46)$$

From the cost function of hospital unit 2 one can deduce that $a < 431,10$, since $C_{2j}^* > 0$ must apply.

After carrying out the calculation of the equilibrium values for the pre- and post-merger situation, a comparison between these two situations is given below.

6.1.3 Comparison between these two Situations

Having calculated the equilibrium values in both situations, a comparison between the outcomes can now be drawn. The investigation of the changes in costs and profits caused by a merger is of particular interest for this purpose. The following table summerizes the results obtained so far:

Before the Merger	After the Merger
$q_{11}^* = q_{12}^* = 0,2550 a - 0,6220$	$q_{11}^* = q_{12}^* = 0,3094 a - 0,1562$
$q_{21}^* = q_{22}^* = 0,2460 a + 0,0363$	$q_{21}^* = q_{22}^* = 0,3800 a - 0,2758$
$q_{31}^* = q_{32}^* = 0,2460 a + 0,0363$	
$Q^* = 1,4940 a - 1,0981$	$Q^* = 1,3788 a - 0,2391$
$P_j^* = 0,2530 a + 0,5490$	$P_j^* = 0,3106 a + 0,4320$

⁷⁸ It is assumed that the number of staffed beds is 30 as in the pre-merger situation.

⁷⁹ It is supposed that for hospital unit 2 the number of staffed beds after the merger is two times larger than before the merger, i.e. 60.

Before the Merger	After the Merger
$C_{1j}^* = -0,0001 a^2 + 0,6033 a + 3.898,04$	$C_{1j}^* = 0,0006 a^2 + 0,3574 a + 4.879,43$
$C_{2j}^* = 0,0017 a^2 + 0,2540 a + 5.655,13$	$C_{2j}^* = -0,0264 a^2 + 0,5368 a + 4.675,01$
$C_{3j}^* = 0,0017 a^2 + 0,2540 a + 5.655,13$	
$\pi_1 = 0,1291 a^2 - 0,6381 a - 3.898,04$	$\pi_1 = 0,1916 a^2 - 0,1871 a - 4.879,43$
$\pi_2 = 0,1228 a^2 + 0,0344 a - 5.655,13$	$\pi_2 = 0,2625 a^2 - 0,3798 a - 4.675,01$
$\pi_3 = 0,1228 a^2 + 0,0344 a - 5.655,13$	

Table 7 Comparison of the Equilibrium Values – Homogenous Outputs

By comparing the output quantities it can be seen, that hospital unit 1 is producing more after the merger than before, whereas hospital ward 2 produces less after the merger than hospital wards 2 and 3 in the pre-merger situation. The table above also shows that the total output in the hospital market is lower in the post-merger situation if $a > 7,46$. Prices will be higher after the merger than before if $a > 2,03$. These results indicate that if $a \in (2,03; 7,46)$ the prices will be higher after the merger even if the total output is higher than before. That is a contradiction to the traditional horizontal merger analysis in chapter 2 where it is stated, that higher output quantities inevitably lead to lower prices.

Moreover, these results indicate that if $a \in (0; 2,03)$ lower prices and higher total output after the merger occur. Nevertheless, it can reasonably be assumed that a is about 7,46 since the number of cases treated or the number of patient days will remain almost constant every year⁸⁰. This means that the prices will definitely be higher after a merger than before.

The costs of hospital unit 1 are higher after the merger if the following inequality holds:

⁸⁰ However it can be assumed that the total output will slightly decrease after a merger, since patients without health insurance (e.g. about 16% in the USA [U.S. Department of Health and Human Services; 2004]) might hesitate to enlist medical assistance at higher prices.

$$C_{1j,post-merger}^* > C_{1j,pre-merger}^*$$

$$0,0006 a^2 + 0,3574 a + 4.879,43 > -0,0001 a^2 + 0,6033 a + 3.898,04$$

$$0,0007 a - 0,2459 + \frac{981,39}{a} > 0 \quad (6.47)$$

Due to the fact that $a \in (0; 431,10)$ this inequality is fulfilled, which means that the costs for hospital ward 1 are always higher after the merger. This is certainly not surprising since hospital unit 1 produces more in the post-merger situation. It is also not involved into a merger hence no efficiency gains can be obtained from that aspect.

The situation differs for hospital ward 2, where efficiency gains can be achieved due to the merger. Comparing the costs of hospital unit 2 after the merger with the costs of hospital units 2 and 3 before the merger one obtains:

$$C_{2j,post-merger}^* > C_{2j,pre-merger}^* + C_{3j}^*$$

$$-0,0264 a^2 + 0,5368 a + 4.675,01 > (0,0017 a^2 + 0,2540 a + 5.655,13) \cdot 2$$

$$-0,0298 a^2 + 0,0288 a - 6.635,25 > 0 \quad (6.48)$$

Since this inequality is never fulfilled the costs after a merger are always lower for hospital unit 2 than the costs before the merger for hospital units 2 and 3. However, this is not surprising since hospital ward 2 produces less after the merger than hospital wards 2 and 3 before the merger. Nevertheless, the coefficients estimated by Sinay indicate efficiency gains⁸¹ being so high, that the costs of hospital unit 2 after the merger are always lower than the costs of hospital unit 2 or 3 before the merger, even if hospital ward 2 after the merger produces more than hospital ward 2 or 3 before the merger.

By comparing the profits of hospital unit 1 it can be observed, that the profits after the merger are higher than before if $a > 121,75$, which is not expected to be the case. This situation is different for hospital unit 2, where the profits are always higher after the merger than the combined profits of hospital wards 2 and 3 before the merger. As mentioned above, this results from the high efficiency gains due to a merger as estimated by Sinay. It should be noted, that if a is about 7,46, the profits obtained by the hospital wards are negative.

6.1.4 Conclusion

The calculation results so far show that a merger between two hospital wards most likely leads to higher prices. It should also be noted that the profits of the merging

⁸¹ Before the merger the estimated coefficients for the output categories squared are positive, whereas after the merger the coefficients turned out to be negative for merging hospitals. These terms are the reason why the costs after the merger are that low.

hospital units are always higher after the merger since the costs are always lower. For the hospital unit which is not involved into a merger the situation is different. It bears higher costs as a result of higher production combined with profits, which only increase after a merger if $a > 121,75$. Since a is expected to be about 7,46 the profits of hospital ward 1 will be lower after a merger than before.

However, a price increase due to a merger can only occur in the private hospital market or for people without a health insurance. This results from the fact that in the private hospital market every hospital negotiates its prices separately with each private insurer in regular intervals, for example once a year. A price increase is also possible for uninsured persons since they have to pay for every treatment at their own expense directly to the hospital.

The situation for the public hospital market, where a public health insurance system is always involved, is fundamentally different. There increasing prices due to a merger are not possible, because the public health insurers have set up their own rules to pay hospitals⁸².

In this context it becomes a necessity to check if the profits of the remaining hospital wards are higher after a merger than before if the prices are not able to change (i.e. the pre-merger prices also occur in the post-merger situation).

For hospital unit 1 this is the case if the following condition is fulfilled:

$$0,0273 a^2 + 0,5413 a - 981,39 > 0 \quad (6.49)$$

This inequality is only valid if $a > 179,95$. Since it is assumed that a is about 7,46 this inequality can not apply. This means that if the prices after the merger are not allowed to change, then the profits of hospital ward 1 are always lower after the merger than before.

In addition, one has to check if the profits of hospital ward 2 are higher after the merger –at pre-merger prices – than the profits of hospital wards 2 and 3 before the merger. The calculation shows that, because of the high cost savings due to the merger, this is the case.

Thus, one can conclude that mergers in the public hospital sector are beneficial to the government if the joint industry profits after a merger are higher than the joint profits before the merger. As stated above, in both cases only the prices of the pre-merger situation can be obtained. This is the case if the following inequality applies:

$$0,0004 a^2 + 0,2134 a + 5.653,86 > 0 \quad (6.50)$$

Since this condition is always fulfilled, it can be noted that even if the profits of hospital unit 1 are lower after the merger, the joint industry profits are higher. This means that, from a financial point of view, mergers between hospital wards are

⁸² See Reinhardt U.E.: The Pricing of U.S. Hospital Services: Chaos Behind a Veil of Secrecy (2006)

always beneficial in the public hospital market because even if the profits of hospital unit 1 are lower after a merger, the total industry profits will rise. Thus, one can conclude that in a public hospital market, where all hospital wards belong to the same owner, a merger is desirable.

In a private hospital market mergers are only beneficial for merging hospital units but not for control hospital units because $a > 121,75$ will not apply. The fact that the industry profits are higher after a merger than before is not relevant for a private hospital market, since both hospital types do not belong to the same owner. A price increase is also not beneficial for uninsured patients, who have to pay medical treatment directly to the hospital.

The results obtained so far depend of course on the shape of the hospital cost functions used, i.e. on the coefficients estimated by Sinay.

After having investigated the consequences of a merger between two hospital wards, the next section will deal with a merger between two hospitals.

6.2 Merger between Hospitals

In the last section a merger between two hospital wards which produce homogeneous outputs was analyzed. In this section a merger between two hospitals with each hospital in the market producing differentiated outputs will be assumed.

The inverse demand function is given by:

$$P_j = a - Q \quad (6.51)$$

In contrast to the linear inverse demand function under (6.3), the price of output j now depends on the quantities of all other outputs and not only on the quantity of output j produced. This assumption is based on the fact that there are no hospitals in the market which produce exactly the same output j . Therefore, the price of output j depends not only on the quantity of output j but also on all other possibly similar outputs. $a > 0$ is again a constant which reflects that the price of output j is positive even if no treatment in this output category occurs.

6.2.1 The Pre-Merger Situation

As in the analysis given in section 6.1 different hospital cost functions for the pre- and post-merger situation have to be assumed. The only difference to the cost function given under (6.5) is that now a merger between two hospitals will result in one new hospital M which produces 4 different output categories.

The hospital cost functions for the pre-merger situation are given below:

- **Cost Function for Merging Hospitals**

- Before the Merger:

$$\begin{aligned}
 C_{ij} = & 0,1637 q_{i1} + 0,1637 q_{i2} + 0,0178 q_{i1}^2 + 0,0178 q_{i2}^2 - 0,0079 q_{i1} q_{i2} \\
 & + 0,6438 w_{i1} + 0,3562 w_{i2} + 0,1762 w_{i1}^2 - 0,0500 w_{i2}^2 - 0,1262 w_{i1} w_{i2} \\
 & + 0,0032 q_{i1} w_{i1} + 0,0032 q_{i2} w_{i1} - 0,0032 q_{i1} w_{i2} - 0,0032 q_{i2} w_{i2} \\
 & + 0,3974 beds
 \end{aligned} \tag{6.52}$$

- **Cost Function for Control Hospitals**

- Before the Merger:

$$\begin{aligned}
 C_{ij} = & 0,1806 q_{i1} + 0,1806 q_{i2} + 0,0065 q_{i1}^2 + 0,0065 q_{i2}^2 - 0,0142 q_{i1} q_{i2} \\
 & + 0,6441 w_{i1} + 0,3559 w_{i2} + 0,1327 w_{i1}^2 - 0,0326 w_{i2}^2 - 0,1001 w_{i1} w_{i2} \\
 & + 0,0096 q_{i1} w_{i1} + 0,0096 q_{i2} w_{i1} - 0,0096 q_{i1} w_{i2} - 0,0096 q_{i2} w_{i2} \\
 & + 0,1954 beds
 \end{aligned} \tag{6.53}$$

The meaning of the different explanatory variables is the same as in section 6.1, i.e.: The variable *beds* represents the number of staffed beds in the hospital and shall reflect the fixed capital. The variable w_{i1} describes the average salary. w_{i2} denotes the average price of supplies for hospital i . q_{i1} and q_{i2} are the two possible outputs every hospital produces.

It is assumed that a merger between hospitals 2 and 3 takes place, with hospital 1 as a control hospital, and hospitals 2 and 3 being the merging hospitals.

Due to the fact that Cournot competition is assumed in the hospital market, each hospital has to maximize its profit function with respect to the quantities it is going to produce. Therefore, every hospital has to maximize π_i :

$$\max. \pi_i = (a - \sum_{i=1}^3 \sum_{j=1}^2 q_{ij}) q_{i1} + (a - \sum_{i=1}^3 \sum_{j=1}^2 q_{ij}) q_{i2} - C_{ij} \tag{6.54}$$

The first order conditions are as follows (note that hospital 1 is a control hospital, and hospitals 2 and 3 are merging hospitals):

- **Hospital 1:**

$$\begin{aligned}
 \frac{\partial \pi_1}{\partial q_{11}} = & a - 2q_{11} - 2q_{12} - q_{21} - q_{22} - q_{31} - q_{32} - 0,1806 - 0,0013 q_{11} \\
 & + 0,0142 q_{12} - 0,0096 w_{11} + 0,0096 w_{12} = 0
 \end{aligned} \tag{6.55}$$

$$\begin{aligned} \frac{\partial \pi_1}{\partial q_{12}} = & a - 2q_{11} - 2q_{12} - q_{21} - q_{22} - q_{31} - q_{32} - 0,1806 - 0,0013 q_{12} \\ & + 0,0142 q_{11} - 0,0096 w_{11} + 0,0096 w_{12} = 0 \end{aligned} \quad (6.56)$$

- **Hospital 2:**

$$\begin{aligned} \frac{\partial \pi_2}{\partial q_{21}} = & a - 2q_{21} - 2q_{22} - q_{11} - q_{12} - q_{31} - q_{32} - 0,1637 - 0,0036 q_{21} \\ & + 0,0079 q_{22} - 0,0032 w_{21} + 0,0032 w_{22} = 0 \end{aligned} \quad (6.57)$$

$$\begin{aligned} \frac{\partial \pi_2}{\partial q_{22}} = & a - 2q_{22} - 2q_{21} - q_{11} - q_{12} - q_{31} - q_{32} - 0,1637 - 0,0036 q_{22} \\ & + 0,0079 q_{21} - 0,0032 w_{21} + 0,0032 w_{22} = 0 \end{aligned} \quad (6.58)$$

- **Hospital 3:**

$$\begin{aligned} \frac{\partial \pi_3}{\partial q_{31}} = & a - 2q_{31} - 2q_{32} - q_{11} - q_{12} - q_{21} - q_{22} - 0,1637 - 0,0036 q_{31} \\ & + 0,0079 q_{32} - 0,0032 w_{31} + 0,0032 w_{32} = 0 \end{aligned} \quad (6.59)$$

$$\begin{aligned} \frac{\partial \pi_3}{\partial q_{32}} = & a - 2q_{32} - 2q_{31} - q_{11} - q_{12} - q_{21} - q_{22} - 0,1637 - 0,0036 q_{32} \\ & + 0,0079 q_{31} - 0,0032 w_{31} + 0,0032 w_{32} = 0 \end{aligned} \quad (6.60)$$

To solve these first order conditions, it is assumed that each hospital in the market produces the same amount of its two different outputs, i.e.:

$$q_{i1} = q_{i2} \quad (6.61)$$

Therefore, after some calculations the equilibrium quantities can be obtained:

$$\begin{aligned} q_{11}^* = q_{12}^* = & 0,1254 a - 0,0024 w_{11} + 0,0024 w_{12} - 0,0008 w_{21} + 0,0008 w_{22} \\ & + 0,0004 w_{31} - 0,0004 w_{32} - 0,0269 \end{aligned} \quad (6.62)$$

$$\begin{aligned} q_{21}^* = q_{22}^* = & 0,1249 a + 0,0008 w_{11} - 0,0008 w_{12} - 0,0008 w_{21} + 0,0008 w_{22} \\ & + 0,0004 w_{31} - 0,0004 w_{32} - 0,0183 \end{aligned} \quad (6.63)$$

$$\begin{aligned} q_{31}^* = q_{32}^* = & 0,1249 a + 0,0008 w_{11} - 0,0008 w_{12} + 0,0008 w_{21} - 0,0008 w_{22} \\ & - 0,0012 w_{31} + 0,0012 w_{32} - 0,0183 \end{aligned} \quad (6.64)$$

The total output in the hospital market is as follows:

$$Q^* = 2 \cdot q_{1j}^* + 2 \cdot q_{2j}^* + 2 \cdot q_{3j}^* \quad (6.65)$$

$$Q^* = 0,7504 a - 0,0016 w_{11} + 0,0016 w_{12} - 0,0016 w_{21} + 0,0016 w_{22} - 0,0008 w_{31} + 0,0008 w_{32} - 0,1270 \quad (6.66)$$

The equilibrium price of output j is given by:

$$P_j^* = a - Q^* \quad (6.67)$$

$$P_j^* = 0,2496 a + 0,0016 w_{11} - 0,0016 w_{12} + 0,0016 w_{21} - 0,0016 w_{22} + 0,0008 w_{31} - 0,0008 w_{32} + 0,1270 \quad (6.68)$$

For a better comparison between the pre- and post-merger situation and furthermore for an easier calculation of the cost and profit functions, concrete values for w_{i1} , w_{i2} and *beds* will be used. Sinay⁸³ states that the mean number of staffed beds is 211 for merging hospitals and 212 for control hospitals. The values for w_{i1} and w_{i2} are shown under (6.23).

Therefore, all equilibrium values calculated so far can be written as follows:

$$\begin{aligned} q_{11}^* &= q_{12}^* = 0,1254 a - 0,3225 \\ q_{21}^* &= q_{22}^* = 0,1249 a + 0,0218 \\ q_{31}^* &= q_{32}^* = 0,1249 a + 0,0218 \end{aligned} \quad (6.69)$$

$$Q^* = 0,7504 a - 0,5579 \quad (6.70)$$

$$P_j^* = 0,2496 a + 0,5579 \quad (6.71)$$

Using the values under (6.23) and the average number of staffed beds for merging and control hospitals, the cost and profit functions for the three hospitals in the market can easily be calculated:

$$\begin{aligned} C_{1j}^* &= 0,2979 a + 3.934,31 \\ C_{2j}^* &= 0,0005 a^2 + 0,1287 a + 5.727,04 \\ C_{3j}^* &= 0,0005 a^2 + 0,1287 a + 5.727,04 \end{aligned} \quad (6.72)$$

⁸³ Sinay U.T.: Pre- and Post-Merger Investigation of Hospital Mergers (1998)

$$\begin{aligned}\pi_1 &= 0,0626 a^2 - 0,3190 a - 3.933,95 \\ \pi_2 &= 0,0619 a^2 + 0,0215 a - 5.727,02 \\ \pi_3 &= 0,0619 a^2 + 0,0215 a - 5.727,02\end{aligned}\quad (6.73)$$

After the investigation of the pre-merger situation, a merger between hospitals 2 and 3 will be assumed.

6.2.2 The Post-Merger Situation

As mentioned at the beginning of this chapter, a merger between hospitals 2 and 3 will lead to a formation of a new hospital M which produces 4 different outputs. This means that for the further analysis only hospital 1, which produces 2 different outputs, and hospital M , which produces 4 different outputs, operate in the market.

Thus, the output matrix of the hospital market is given by:

$$Q = \begin{bmatrix} q_{11} & q_{12} & 0 & 0 \\ q_{M1} & q_{M2} & q_{M3} & q_{M4} \end{bmatrix} \quad (6.74)$$

In the further analysis $i = 1, M$ denotes the two hospitals, and $j = 1, 2, 3, 4$ the different outputs in the market.

The cost functions for the post-merger situation are given below:

- **Cost Function for Control Hospitals (i.e. Hospital 1)**

- After the Merger:

$$\begin{aligned}C_{ij} &= 0,1720 q_{i1} + 0,1720 q_{i2} + 0,0066 q_{i1}^2 + 0,0066 q_{i2}^2 - 0,0069 q_{i1} q_{i2} \\ &\quad + 0,6382 w_{i1} + 0,3618 w_{i2} + 0,1541 w_{i1}^2 - 0,0662 w_{i2}^2 - 0,0879 w_{i1} w_{i2} \\ &\quad + 0,0039 q_{i1} w_{i1} + 0,0039 q_{i2} w_{i1} - 0,0039 q_{i1} w_{i2} - 0,0039 q_{i2} w_{i2} \\ &\quad + 0,2272 \text{ beds}\end{aligned}\quad (6.75)$$

- **Cost Function for Merging Hospitals (i.e. Hospital M)**

- After the Merger:

$$\begin{aligned}C_{ij} &= 0,2777 q_{i1} + 0,2777 q_{i2} + 0,2777 q_{i3} + 0,2777 q_{i4} - 0,0981 q_{i1}^2 \\ &\quad - 0,0981 q_{i2}^2 - 0,0981 q_{i3}^2 - 0,0981 q_{i4}^2 + 0,0136 q_{i1} q_{i2} + 0,0136 q_{i1} q_{i3} \\ &\quad + 0,0136 q_{i1} q_{i4} + 0,0136 q_{i2} q_{i3} + 0,0136 q_{i2} q_{i4} + 0,0136 q_{i3} q_{i4} \\ &\quad + 0,6381 w_{i1} + 0,3619 w_{i2} + 0,1762 w_{i1}^2 - 0,0033 w_{i2}^2 - 0,1796 w_{i1} w_{i2} \\ &\quad + 0,0035 q_{i1} w_{i1} + 0,0035 q_{i2} w_{i1} + 0,0035 q_{i3} w_{i1} + 0,0035 q_{i4} w_{i1} \\ &\quad - 0,0035 q_{i1} w_{i2} - 0,0035 q_{i2} w_{i2} - 0,0035 q_{i3} w_{i2} - 0,0035 q_{i4} w_{i2} \\ &\quad - 0,2529 \text{ beds}\end{aligned}\quad (6.76)$$

It is assumed that the two hospitals in the market compete in quantities. For that reason they have to maximize their profit functions in order to find the optimal quantities of the different outputs. The profit functions for hospital 1 and hospital M are given as follows:

$$\pi_1 = (a - Q) q_{11} + (a - Q) q_{12} - C_{1j}$$

$$\pi_M = (a - Q) q_{M1} + (a - Q) q_{M2} + (a - Q) q_{M3} + (a - Q) q_{M4} - C_{Mj} \quad (6.77)$$

To obtain the equilibrium output levels, the first order conditions have to be calculated (note that hospital 1 is a control hospital, and hospital M is a merging hospital):

- **Hospital 1:**

$$\begin{aligned} \frac{\partial \pi_1}{\partial q_{11}} = & a - 2q_{11} - 2q_{12} - q_{M1} - q_{M2} - q_{M3} - q_{M4} - 0,1806 - 0,0013 q_{11} \\ & + 0,0142 q_{12} - 0,0096 w_{11} + 0,0096 w_{12} = 0 \end{aligned} \quad (6.78)$$

$$\begin{aligned} \frac{\partial \pi_1}{\partial q_{12}} = & a - 2q_{11} - 2q_{12} - q_{M1} - q_{M2} - q_{M3} - q_{M4} - 0,1806 - 0,0013 q_{12} \\ & + 0,0142 q_{11} - 0,0096 w_{11} + 0,0096 w_{12} = 0 \end{aligned} \quad (6.79)$$

- **Hospital M:**

$$\begin{aligned} \frac{\partial \pi_2}{\partial q_{M1}} = & a - 2q_{M1} - 2q_{M2} - 2q_{M3} - 2q_{M4} - q_{11} - q_{12} - 0,2777 + 0,1962 q_{M1} \\ & - 0,0136 q_{M2} - 0,0136 q_{M3} - 0,0136 q_{M4} - 0,0035 w_{M1} + 0,0035 w_{M2} = 0 \end{aligned} \quad (6.80)$$

$$\begin{aligned} \frac{\partial \pi_2}{\partial q_{M2}} = & a - 2q_{M1} - 2q_{M2} - 2q_{M3} - 2q_{M4} - q_{11} - q_{12} - 0,2777 + 0,1962 q_{M2} \\ & - 0,0136 q_{M1} - 0,0136 q_{M3} - 0,0136 q_{M4} - 0,0035 w_{M1} + 0,0035 w_{M2} = 0 \end{aligned} \quad (6.81)$$

$$\begin{aligned} \frac{\partial \pi_2}{\partial q_{M3}} = & a - 2q_{M1} - 2q_{M2} - 2q_{M3} - 2q_{M4} - q_{11} - q_{12} - 0,2777 + 0,1962 q_{M3} \\ & - 0,0136 q_{M1} - 0,0136 q_{M2} - 0,0136 q_{M4} - 0,0035 w_{M1} + 0,0035 w_{M2} = 0 \end{aligned} \quad (6.82)$$

$$\begin{aligned} \frac{\partial \pi_2}{\partial q_{M4}} = & a - 2q_{M1} - 2q_{M2} - 2q_{M3} - 2q_{M4} - q_{11} - q_{12} - 0,2777 + 0,1962 q_{M4} \\ & - 0,0136 q_{M1} - 0,0136 q_{M2} - 0,0136 q_{M3} - 0,0035 w_{M1} + 0,0035 w_{M2} = 0 \end{aligned} \quad (6.83)$$

Again, it is assumed that every hospital produces the same amount of their different outputs, i.e.:

$$\begin{aligned} q_{11} &= q_{12} \\ q_{M1} &= q_{M2} = q_{M3} = q_{M4} \end{aligned} \quad (6.84)$$

After solving the first order conditions, the equilibrium output levels can be obtained:

$$\begin{aligned} q_{11}^* = q_{12}^* = & 0,1650 a - 0,0031 w_{11} + 0,0031 w_{12} \\ & + 0,0006 w_{M1} - 0,0006 w_{M2} - 0,0133 \end{aligned} \quad (6.85)$$

$$\begin{aligned} q_{M1}^* = q_{M2}^* = q_{M3}^* = q_{M4}^* = & 0,0854 a + 0,0008 w_{11} - 0,0008 w_{12} \\ & - 0,0006 w_{M1} + 0,0006 w_{M2} - 0,0320 \end{aligned} \quad (6.86)$$

The total output in the hospital market is given by:

$$Q^* = 2 \cdot q_{1j}^* + 4 \cdot q_{Mj}^* \quad (6.87)$$

$$\begin{aligned} Q^* = & 0,6716 a - 0,0030 w_{11} + 0,0030 w_{12} \\ & - 0,0012 w_{M1} + 0,0012 w_{M2} - 0,1546 \end{aligned} \quad (6.88)$$

The equilibrium price of output j in the post-merger situation is as follows:

$$P_j^* = a - Q^* \quad (6.89)$$

$$\begin{aligned} P_j^* = & 0,3284 a + 0,0030 w_{11} - 0,0030 w_{12} \\ & + 0,0012 w_{M1} - 0,0012 w_{M2} + 0,1546 \end{aligned} \quad (6.90)$$

Again, using the concrete values for the average salary and the average supply price under (6.23)⁸⁴, the equilibrium values calculated above can be rewritten:

$$\begin{aligned} q_{11}^* = q_{12}^* &= 0,1650 a - 0,2727 \\ q_{M1}^* = q_{M2}^* = q_{M3}^* = q_{M4}^* &= 0,0854 a - 0,0138 \end{aligned} \quad (6.91)$$

⁸⁴ $w_{11} = \$ 233,9$; $w_{12} = \$ 129$ and $w_{M1} = \$ 243,6$; $w_{M2} = \$ 134$

$$Q^* = 0,6716 a - 0,6008 \quad (6.92)$$

$$P_j^* = 0,3284 a + 0,6008 \quad (6.93)$$

Finally, the cost and profit functions from the two hospitals in the market have to be calculated. Therefore, it is assumed that the number of staffed beds is 212 for hospital 1 and 422 for hospital M ⁸⁵.

$$\begin{aligned} C_{1j}^* &= 0,0002 a^2 + 0,1912 a + 4.920,64 \\ C_{Mj}^* &= -0,0023 a^2 + 0,2266 a + 4.631,22 \end{aligned} \quad (6.94)$$

$$\begin{aligned} \pi_1 &= 0,1082 a^2 - 0,1720 a - 4.920,97 \\ \pi_M &= 0,1145 a^2 - 0,0395 a - 4.631,25 \end{aligned} \quad (6.95)$$

The cost function from hospital M has to be positive, which means that $a < 1.469,12$ must apply.

After calculating various equilibrium values for the pre- and post-merger situation, a comparison between these results can be given below.

6.2.3 Comparison between these two Situations

Due to a better comparison of the numerous equilibrium values calculated, the following table shall summarize the obtained results:

Before the Merger	After the Merger
$q_{11}^* = q_{12}^* = 0,1254 a - 0,3225$	$q_{11}^* = q_{12}^* = 0,1650 a - 0,2727$
$q_{21}^* = q_{22}^* = 0,1249 a + 0,0218$	$q_{M1}^* = q_{M2}^* = q_{M3}^* = q_{M4}^* = 0,0854 a - 0,0138$
$q_{31}^* = q_{32}^* = 0,1249 a + 0,0218$	
$Q^* = 0,7504 a - 0,5579$	$Q^* = 0,6716 a - 0,6008$

⁸⁵ As noted in the pre-merger situation Sinay states that the average number of staffed beds is 211 for merging hospitals and 212 for control hospitals. Therefore, it is assumed that hospital 1 has 212 staffed beds and hospital M has $2 \cdot 211 = 422$ staffed beds.

Before the Merger	After the Merger
$P_j^* = 0,2496 a + 0,5579$	$P_j^* = 0,3284 a + 0,6008$
$C_{1j}^* = 0,2979 a + 3.934,31$	$C_{1j}^* = 0,0002 a^2 + 0,1912 a + 4.920,64$
$C_{2j}^* = 0,0005 a^2 + 0,1287 a + 5.727,04$	$C_{Mj}^* = -0,0023 a^2 + 0,2266 a + 4.631,22$
$C_{3j}^* = 0,0005 a^2 + 0,1287 a + 5.727,04$	
$\pi_1 = 0,0626 a^2 - 0,3190 a - 3.933,95$	$\pi_1 = 0,1082 a^2 - 0,1720 a - 4.920,97$
$\pi_2 = 0,0619 a^2 + 0,0215 a - 5.727,02$	$\pi_M = 0,1145 a^2 - 0,0395 a - 4.631,25$
$\pi_3 = 0,0619 a^2 + 0,0215 a - 5.727,02$	

Table 8 Comparison of the Equilibrium Values – Differentiated Outputs

By comparing the output quantities it can be seen that hospital 1 treats more cases or rather patient days after the merger than before, whereas hospital M produces less than hospitals 2 and 3 combined before the merger. The total output in the hospital market is lower after the merger than before if $a > -0,54$. This means that the total output in the hospital market is always lower after a merger since it is assumed that a is always positive. The price of output j after the merger will rise if $a > -0,54$, which practically always applies. Furthermore, all output quantities in the pre- and post-merger situation are only positive if $a > 2,57$. As mentioned in the analysis of hospital ward mergers given above, it can be expected that the total output in the hospital market will not change significantly due to a merger since the number of cases treated or the number of patient days will remain almost constant every year. Thus, it is assumed that a equals approximately 2,6 because then the total output before and after a merger will be roughly the same (1,39 units of output before the merger and 1,15 units of output after the merger). Moreover, this value for a results in higher post-merger prices (\$ 1,21 before the merger and \$ 1,45 after the merger).

Comparing the costs of hospital 1 for the pre- and post-merger situation, $C_{1j,post-merger}^* > C_{1j,pre-merger}^*$ applies if:

$$0,0002 a^2 + 0,1912 a + 4.920,64 > 0,2979 a + 3.934,31$$

$$0,0002 a - 0,1067 + \frac{986,33}{a} > 0 \quad (6.96)$$

Since $a \in (2,57; 1.496,12)$ this inequality is always fulfilled. This means that the costs of hospital 1 are higher after the merger since the hospital is producing more but cannot obtain any further efficiency gains from joint production. In contrast, the efficiency gains in the pre-merger situation are higher as a result of the joint production, than they are in the post-merger situation⁸⁶.

The cost situation is different for hospital M because high efficiency gains owing to the merger can be obtained. By comparing the cost functions of the pre-merger and the post-merger situation it can be seen, that the coefficient from the variable q_{Mj}^2 changed significantly: From +0,0178 before the merger to -0,0981 after the merger. However, it should be tested whether the costs of hospital M are higher for some values of a than the costs for hospitals 2 and 3:

$$C_{Mj}^* > C_{2j}^* + C_{3j}^*$$

$$-0,0023 a^2 + 0,2266 a + 4.631,22 > (0,0005 a^2 + 0,1287 a + 5.727,04) \cdot 2$$

$$-0,0033 a^2 - 0,0308 a - 6.822,86 > 0 \quad (6.97)$$

Since hospital M produces less than hospitals 2 and 3 and moreover high efficiency gains due to the merger can be obtained the inequality under (6.97) is never fulfilled. Hence the costs for hospital M are always lower than for hospitals 2 and 3 combined. Furthermore, the efficiency gains estimated by Sinay are so high, that the costs of hospital M are lower than the sole costs of each individual hospital 2 or 3 respectively.

With regards to the profit functions in the equilibrium it can be said, that if $a > 145,52$ the profits of hospital 1 are higher after the merger than before. Since it can be expected that the total output in the hospital market will not change significantly (i.e. a is about 2,6), the profits of hospital 1 will decrease due to the merger.

By comparing the profits of hospital M with those of hospitals 2 and 3 it can be seen that the profits of hospital M are higher if $a < 852,09$. The application of the above explanation (a is about 2,6) indicates, that the profits of hospital M are higher than those of hospitals 2 and 3.

Moreover, if a is about 2,6, the profits realized by the hospitals are negative in both situations.

⁸⁶ The coefficient from the interaction term $q_{i1} q_{i2}$ is -0,0142 in the pre-merger situation, and -0,0069 in the post-merger situation.

6.2.4 Conclusion

As demonstrated in this section a merger between two hospitals will always lead to higher prices and lower industry output. The calculation results also show that the profits of the merging hospital M are higher after the merger if $a < 852,09$. Since the total industry output before and after the merger remains approximately the same (a is about 2,6) this will always be the case. The higher profits are caused by the higher prices but also the high efficiency gains achievable if a merger takes place. The situation for hospital 1 is different. The profits are only higher in the post-merger situation if $a > 145,52$ which is presumably not the case. Furthermore, hospital 1's costs will always be higher after hospitals 2 and 3 have merged.

Nevertheless, higher prices are only possible in the private hospital market where each individual hospital negotiates its prices separately with the private health insurers in regular intervals. Price increases are also possible for uninsured persons who have to pay a direct bill from the hospital for every treatment at their own expense. In contrast, higher prices are not feasible in the public hospital market since there public health insurers, which have their own rules for paying the hospitals, are always involved⁸⁷.

Thus it has to be investigated whether the total profits in the public hospital market are higher after a merger if only the pre-merger prices can be obtained. It can be expected that the profits of hospital 1 are lower after a merger than before since $a > 145,52$ will not apply. Thus, it is obvious that the profits of hospital 1 are still lower after the merger if the lower price from the pre-merger situation will be included in the calculation.

The question whether or not the efficiency gains due to the merger are so high, that the profits of hospital M after the merger (based on pre-merger prices) are higher than the profits of hospitals 2 and 3 before the merger, is of particular interest. This applies if the following condition is fulfilled:

$$-0,0368 a^2 - 0,0928 a + 6.822,79 > 0 \quad (6.98)$$

The inequality under (6.98) is valid if $a < 429,32$. Since the total output in the hospital market is only similar before and after the merger if a is about 2,6, it can be concluded that this condition is fulfilled.

The joint profits in the public hospital market are higher after the merger if the following inequality is correct:

$$-0,0166 a^2 + 0,083 a + 5.835,80 > 0 \quad (6.99)$$

Condition (6.99) applies if $a < 595,43$, which obviously is the case.

The above results show that a merger between public hospitals is beneficial since the joint industry profits are higher after a merger than before and control and merging

⁸⁷ See Reinhardt U.E.: The Pricing of U.S. Hospital Services: Chaos Behind A Veil of Secrecy (2006)

hospitals are both owned by the government. A merger in the private hospital market is only beneficial for merging hospitals but not for control hospitals because $a > 145,52$ will not apply. Furthermore, a merger is not desirable for persons without health insurance, who would have to pay higher prices as an effect of the merger.

In summery it can be stated, that a merger will always be profitable for the merging parties. In contrast the profits of the control hospitals or hospital wards are presumably lower after a merger than before. However, if the control and merging hospitals/hospital wards are public ones, the higher profits of the merging hospitals will more than outweigh the lower profits of the control hospitals. This does not apply to the private hospital market, since the control and merging hospitals/hospital wards do not belong to the same owner. Therefore, it can be expected that in the private hospital market control hospitals will oppose a planned merger.

Nevertheless, I would like to note that these results depend on the shape of the hospital cost functions used for the calculations. In this case the results depend on the coefficients estimated by Sinay⁸⁸.

However, as mentioned in chapter 5, the selection of a representative cost function is a delicate issue: While several authors have estimated hospital cost functions, almost none of them has estimated hospital cost functions before and after a merger. Thus, it has to be assumed that the coefficients obtained by Sinay are characteristic, especially if looking at the high efficiency gains achieved by merging hospitals due to a merger.

⁸⁸ Sinay U.T.: Pre- and Post-Merger Investigation of Hospital Mergers (1998)

7 Conclusion

7.1 Summary of the Results

The merger analysis of the hospital sector shows that the costs for control hospitals or hospital wards are higher after a merger than before⁸⁹. However, this does not apply to merging hospitals or hospital wards, which will benefit from high efficiency gains due to the merger (as estimated by Sinay⁹⁰). Furthermore, the profits of control hospitals or hospital wards will decrease as a result of a merger, whereas they will increase for merging hospitals or hospital wards.

With regard to the prices it is assumed that these will not change as a consequence of a merger in the public hospital market, since both health insurers and hospitals are under public administration and the terms of payment are agreed directly between hospitals and public health insurers. In contrast, the analysis shows that for the private hospital market a price increase is most likely for a merger of two hospital wards whereas it will definitely result from a merger between two hospitals.

Focusing on profits and costs only, the results of the merger analysis⁹¹ indicate that mergers between public hospital wards or public hospitals are always beneficial. This is supported by the fact that the joint profits of the total hospital market are higher after a merger than before, even if the prices remain the same. Thus mergers in a public hospital market can be regarded as an efficient tool for profit maximization and could consequently reduce the financial strain on the public healthcare system.

Above clear statement cannot be given for a private hospital market. Although merging hospital wards or merging hospitals will benefit from a merger, all other market participants will suffer financial losses. This could lead to the closure of less profitable hospitals and thus cause a reduced density of supply with medical service for the public. Therefore, a recommendation of a planned merger will depend first and foremost on the division of the hospital market between private and public healthcare providers.

7.2 Discussion

The thesis shows that analyzing the hospital market several aspects, which play an essential role for the reliability and accuracy of the results, must be considered. With regard to the question how to measure hospital outputs it can be said, that the number of cases treated is the most defensible unit of output. This statement is supported by the fact, that modern terms of payment for hospitals are usually

⁸⁹ This applies under the assumption that the total industry output does not differ significantly before and after a merger. This seems to be realistic since the number of cases treated or the number of patient days will remain nearly constant every year, regardless of whether a merger has taken place or not.

⁹⁰ Sinay U.T.: Pre- and Post-Merger Investigation of Hospital Mergers (1998)

⁹¹ Generally, the results obtained depend on the shape of the hospital cost function used for calculation. Considering this it can be said, that all cost functions analyzed in the present paper show substantial similarity in shape.

performance-orientated⁹². Notwithstanding above said, none of the authors who have estimated hospital cost functions based on reliable empirical data has used the number of cases treated as the unit of output. Since these hospital data are not always easily available, the number of patient days is usually the only left output choice to estimate these functions. Thus significant output coefficients based on patient days had to be used for the horizontal merger analysis in the present work⁹³.

As a consequence of the lack of scientific literature on hospital cost functions in Europe, coefficients estimated by Sinay⁹⁴ were used for the analysis. These estimations are based on data from hospitals in the United States for the time period 1987 – 1989. In order to determine whether the results obtained can also be applied to a typical hospital market in the European Union, a brief comparison between the hospital markets in the United States and for example Germany, having the largest population within the EU, is given below.

With regard to the ownership type in the U.S. it can be said, that in the year 1999 24% of all hospitals were owned by the state/local government, whereas 61% were private non-profit hospitals. The remaining 15% of all hospitals in the U.S. were private for-profit ones⁹⁵.

On the other hand Wörz & Busse⁹⁶ note that in the year 2001 53,6% of all German acute hospital beds were publicly owned, whereas 38,4% of all acute hospital beds belonged to private not-for-profit acute hospitals. Only 8% of all acute hospital beds in Germany were provided by private for-profit acute hospitals.

While in the U.S. (1999) 76% of all hospitals were privately owned, only 46,4% of all acute German hospitals (2001) belonged to the private hospital market. This indicates that a price increase due to a merger seems more likely in the U.S. compared to Germany, since in the public hospital market a public health insurance system, with fixed compensation arrangements for hospitals, is involved.

With regard to the costs Anderson & Reinhardt & Hussey & Petrosyan⁹⁷ note, that in the year 2000 only 3 acute care beds per 1.000 population in the U.S. were available, that 118 admissions per 1.000 population occurred, that the average length of stay was 5,9 days (for the year 1999), and 0,7 acute care hospital days per capita were present.

In Germany on the other hand, 6,4 acute care beds per 1.000 population were available, 205 admissions per 1.000 population were present, the average length of stay was 9,6 days, and 1,9 acute care hospital days per capita occurred.

⁹² E.g.: Until 1997 the number of patient days has been the unit of measurement for the payment of hospitals in Austria. Since then a performance-orientated paying system based on admission and diagnosis is valid. – [Köck C.: Krankenhäuser können gefährlich sein (2009)]

⁹³ For this calculation, mean values of the estimated coefficients based on empirical data from 202 merging hospitals and from 202 control hospitals from the United States (nationwide sample) for the years 1987 – 1989, 1 year before and 2 years after a merger have been used. – [Sinay U.T.: Pre- and Post-Merger Investigation of Hospital Mergers (1998)]

⁹⁴ Sinay U.T.: Pre- and Post-Merger Investigation of Hospital Mergers (1998)

⁹⁵ See Kaiser Family Foundation: www.statehealthfacts.org

⁹⁶ Wörz M., Busse R.: Analysing the impact of health-care system change in the EU member states – Germany (2005)

⁹⁷ Anderson G.F., Reinhardt U.E., Hussey P.S., Petrosyan V.: It's The Prices, Stupid: Why The United States Is So Different From Other Countries (2003)

Moreover, the authors suggest that though Germany has both more acute care beds and admissions per capita and also a higher average length of stay than the U.S., the average costs per patient day or per hospital admission are considerably lower than those of the U.S. Hence Germany spends less per capita and as a percentage of the *GDP* on hospital care than the U.S. does. Providing an explanation for these differences, Anderson & Reinhardt & Hussey & Petrosyan state that the diverse inputs used for providing hospital care (e.g. salaries, medical equipment, pharmaceuticals) are more expensive in the U.S. than in other countries. Furthermore, they note that the U.S. hospital market could be less efficient compared to other countries because of the complex U.S. payment system, which could require more administrative personnel in U.S. hospitals. In addition, the authors show that in the years 1999 and 2000 sophisticated medical technologies (e.g. magnetic resonance imaging, coronary angioplasties and patients undergoing dialysis) were used more frequently in the U.S. compared to Germany.

In summary it can be stated that the general applicability of hospital cost functions estimated for the U.S. to the EU hospital market can not be confirmed, since the average costs per patient day are substantially higher in the U.S. than in other countries.

Moreover, price increases as a result of a merger are more likely than in the EU since private hospitals hold the largest market share in the U.S.

Since the estimated hospital cost functions for the U.S. form the basis of the present hospital merger analysis, the results obtained would have to be tested before their application to any other hospital market in Europe.

One can conclude, that the lack of general applicability of hospital cost functions based on U.S. data and the use of patient days as unit of output represent the most problematic parts of the present analysis. In order to conduct a reliable and sophisticated merger analysis of the hospital sector in Europe, coefficients derived from empirical data from EU hospitals before and after a merger would be needed, which however were not publicly available for the preparation of the present thesis. Thus more empirical research on hospital mergers in Europe would be highly desirable. Moreover, hospital output data being collected in the future should only be based on the number of cases treated and not on the number of patient days, which may differ enormously between hospitals and do not reflect performance-orientated paying rules.

Abstract

The present master thesis deals with mergers in the hospital market and their consequences, particularly on costs and profits. In the process, mergers between hospital wards and mergers between hospitals are analyzed. Therefore, it is assumed that hospital wards produce homogenous outputs whereas hospitals produce differentiated outputs. Before carrying out the analysis I investigate the questions which outputs a hospital is going to produce and how to measure the units of those. Different hospital cost functions are discussed and the relevant factors to be included in the merger analysis are determined. Since hospitals are multiproduct firms, also a brief reminder about multiproduct cost functions is given. Traditional merger analysis outlines only the consequences of a merger between single product firms. Therefore, the basic conclusions in traditional merger analysis are quoted.

Mean values of the statistically significant coefficients estimated by Sinay⁹⁸ are used to define the hypothesized hospital cost functions required for the analysis. The results⁹⁹ show, that under the assumption that the total industry output does not differ significantly before and after a merger¹⁰⁰, the costs for control hospitals or hospital wards are higher after a merger than before. However, this does not apply to merging hospitals or hospital wards, which will benefit from high efficiency gains due to the merger (as estimated by Sinay). Furthermore, the merger analysis of the hospital sector shows that the profits of control hospitals or hospital wards will decrease as a result of a merger, whereas they will increase for merging hospitals or hospital wards.

With regard to the prices it is assumed that these will not change as a consequence of a merger in the public hospital market, since both health insurers and hospitals are under public administration. Thus the terms of payment are agreed directly between hospitals and public health insurers. In contrast, the analysis shows that for the private hospital market a price increase is most likely for a merger of two hospital wards whereas it will definitely result from a merger between two hospitals.

Moreover, the higher profits of the merging hospitals or hospital wards after a merger can easily balance the loss of profits of those not involved in a merger, even if the prices remain unchanged. This indicates that particularly in the public hospital market mergers are desirable, since all hospital units and hospitals belong to the same owner (i.e. the public authorities).

⁹⁸ Sinay U.T.: Pre- and Post-Merger Investigation of Hospital Mergers (1998)

⁹⁹ Generally, the results obtained depend on the shape of the hospital cost function used for calculation. Considering this it can be said that all cost functions analyzed in the present paper show substantial similarity in shape.

¹⁰⁰ This assumption seems to be realistic, since the number of cases treated or the number of patient days will remain nearly constant every year, regardless of whether a merger has taken place or not.

Zusammenfassung

Die vorliegende Diplomarbeit befasst sich mit Fusionen im Krankenhausmarkt und deren Auswirkungen, speziell auf Kosten und Gewinne. Dabei werden Fusionen zwischen Krankenhausabteilungen und Fusionen zwischen Krankenhäusern analysiert. Es wird angenommen, dass Krankenhausabteilungen homogene Outputs und Krankenhäuser differenzierte Outputs produzieren. Vor Durchführung der Untersuchung muss jedoch die Frage beantwortet werden welche Outputs ein Krankenhaus überhaupt produziert, und in welchen Einheiten diese gemessen werden können. Verschiedene Kostenfunktionen von Krankenhäusern werden diskutiert und die relevanten Faktoren, welche in eine Kostenfunktion inkludiert werden sollten, bestimmt. Da Krankenhäuser Mehrproduktunternehmen darstellen, wird ein kurzer Überblick über Mehrprodukt-Kostenfunktionen gegeben. Traditionell beschäftigt sich die Fusionsanalyse mit den Konsequenzen von Zusammenschlüssen von Ein-Produktunternehmen, weshalb ebenfalls die grundsätzlichen Folgerungen der traditionellen Fusionsanalyse aufgeführt sind.

Zur Festlegung der für die Analyse benötigten hypothetischen Kostenfunktionen werden Mittelwerte der statistisch signifikanten Koeffizienten nach der Abschätzung von Sinay¹⁰¹ herangezogen. Die Resultate¹⁰² zeigen, dass unter der Annahme eines gleichbleibenden Gesamt-Outputs im Krankenhausmarkt vor und nach einer Fusion¹⁰³, die Kosten für die Kontroll-Krankenhäuser bzw. -Abteilungen nach einer Fusion höher sind als zuvor. Dies gilt jedoch nicht für fusionierende Krankenhäuser bzw. Abteilungen, welche von den hohen Effizienzgewinnen in Folge einer Fusion profitieren (wie von Sinay geschätzt). Darüber hinaus zeigt die Fusionsanalyse des Krankenhaussektors, dass die Gewinne der Kontroll-Krankenhäuser bzw. -Abteilungen durch eine Fusion abnehmen, wohingegen sie für die fusionierenden Krankenhäuser und Abteilungen zunehmen werden.

Da im öffentlichen Krankenhausmarkt sowohl Gesundheitsversicherer als auch Spitäler unter öffentlicher Verwaltung stehen kann vorausgesetzt werden, dass sich die Preise aufgrund einer Fusion nicht ändern. Der Grund hierfür ist, dass die Zahlungsbedingungen direkt zwischen Spitälern und öffentlichen Gesundheitsversicherern festgelegt werden. Demgegenüber zeigt die Untersuchung, dass eine Preissteigerung im privaten Krankenhausmarkt als Folge der Fusion zweier Krankenhausabteilungen sehr wahrscheinlich, als Folge einer Fusion zwischen zwei Krankenhäusern jedoch unausweichlich ist.

Darüber hinaus können die höheren Gewinne der fusionierenden Krankenhäuser bzw. Abteilungen die geringeren Profite derjenigen, welche nicht in eine Fusion involviert sind, mehr als aufwiegen. Da im öffentlichen Krankenhausmarkt alle Krankenhausabteilungen bzw. Krankenhäuser im Besitz ein und desselben Eigentümers stehen (d.h. der öffentlichen Hand) kann der Schluss gezogen werden, dass besonders dort Fusionen vorteilhaft sind.

¹⁰¹ Sinay U.T.: Pre- and Post-Merger Investigation of Hospital Mergers (1998)

¹⁰² Die erhaltenen Resultate hängen im Allgemeinen von der Form, der für die Berechnungen verwendeten Spitalskostenfunktionen ab. Dazu kann festgehalten werden, daß alle im Zuge dieser Arbeit untersuchten Kostenfunktionen substantielle Ähnlichkeit in Ihrer Form aufweisen.

¹⁰³ Diese Annahme scheint realistisch zu sein, da die Anzahl der behandelten Fälle oder die Anzahl der Patiententage jedes Jahr ungefähr gleich sein wird, unabhängig davon ob eine Fusion stattgefunden hat oder nicht.

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